PREFACE

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GENERATION OF W-TYPE STATES IN THREE KERR-LIKE NONLINEAR OSCILLATORS SYSTEM

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Abstract: *In this article, we examine a model comprising three Kerr-like nonlinear oscillators with two boundaries that are pumped by external coherent fields and coupled to the center one. We demonstrate that by applying evolution operator formalism, our quantum system can be simulated and behave as a nonlinear quantum scissors, its "truncation" Hilbert space is more efficient. It will be confirmed that the model can produce bipartite and tripartite entanglement, especially W-type entangled states which is a remarkably simple quantum information problem.*

Keywords*: Entanglement, W state qubit, qutrit, negativity, quantum scissors.*

1. Introduction

The journey of Quantum Mechanics began with Planck's proposal of energy quantization and the law of radiation to explain the so-called "ultraviolet catastrophe" in the spectrum of matter. This idea was enhanced one step by Einstein when he did research on the photoelectric effect. With the time, Quantum mechanics has been improved, helping us to understand the framework of physics and its principle, it has also become a basis for advancement of many new branches of physics such as quantum information, teleportation, computation and many other fields. The essential features for such advances are known as quantum correlations and they can be affirmed to transmit, preserve and manipulate information. These led to a considerable development of particular interest in research of quantum correlations in numerous types of quantum systems. The purpose of this article is to investigate how to generate time evolution of quantum correlations in terms of the distinctive form - entanglement. Specifically, the system consists of three nonlinear Kerr-type oscillators, two boundaries are mutually coupled to center one by continuous linear interaction and excited by the external coherent fields. These oscillators can be expressed by effective Hamiltonians which are alike to those described optical Kerr systems. Quantum Kerr-type nonlinearity systems are commonly discussed in various physical applications. Such models can be put into practice in description of nanomechanical resonators and many optomechanical systems with Bose-Einstein condensate [14]. In addition, Kerr-type oscillatory models were the subject of a large number of papers related

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to the quantum chaos problems [7], [15]. The modes of Kerr-type nonlinear coupler have been proven to be auspicious devices, simple treatment for finding numerical solutions and producing entangled states and hence its quantumness.

2. The model description and discussions

The considered system comprises three nonlinear Kerr-type oscillators, they are mutually coupled to each other by linear interaction and excited by external excitation fields in two boundaries, each oscillator corresponds to a single mode of the field labeled *a, b* and *c*. It is not only the self- coupling term that exists in this system, instead of the so-called cross-Kerr coupling involving [9] which is different from many systems in previous literature [6], [7]. The total Hamiltonian describing the dynamics of our system can be defined as (assume that $\hbar = 1$).

$$\hat{H} = \hat{H}_{nl} + \hat{H}_{\text{int}} + \hat{H}_{ext}, \tag{1}$$

where

$$\hat{H}_{nl} = \frac{\chi_a}{2}\hat{a}^{\dagger 2}\hat{a}^2 + \frac{\chi_b}{2}\hat{b}^{\dagger 2}\hat{b}^2 + \frac{\chi_c}{2}\hat{c}^{\dagger 2}\hat{c}^2 + \chi_{ab}\hat{a}^{\dagger}\hat{a}\hat{b}^{\dagger}\hat{b} + \chi_{bc}\hat{b}^{\dagger}\hat{b}\hat{c}^{\dagger}\hat{c} + \chi_{ac}\hat{a}^{\dagger}\hat{a}\hat{c}^{\dagger}\hat{c}, \tag{2}$$

defines Kerr-like media (including cross-Kerr coupling); and

$$\hat{H}_{\text{int}} = \varepsilon_{ab}\hat{a}^{\dagger}\hat{b} + \varepsilon^{*}_{ab}\hat{a}\hat{b}^{\dagger} + \varepsilon_{bc}\hat{b}^{\dagger}\hat{c} + \varepsilon^{*}_{bc}\hat{b}\hat{c}^{\dagger}, \tag{3}$$

corresponds to the nonlinear interaction between coupled modes *a*-*b* and *b*-*c*;

$$\hat{H}_{ext} = \alpha\hat{a}^{\dagger} + \alpha^{*}\hat{a} + \gamma\hat{c}^{\dagger} + \gamma^{*}\hat{c} \tag{4}$$

relates to interaction with external fields.

Here χ_a, χ_b, χ_c are proportional to the third-order susceptibilities; $\chi_{ab}, \chi_{bc}, \chi_{ac}$ describe the cross - Kerr coupling processes between a - b, and b – c; whereas ε_{ab} and ε_{bc} mean the strength of the nonlinear interactions between modes a - b, and b - c. Hamiltonian of our system can be stated in terms of bosonic annihilation and creation operators, we are able to present annihilation operators as square matrices in the Hilbert space $H = H_a \otimes H_b \otimes H_c$ as follows:

$$\widehat{a} = \hat{I}_r \otimes \hat{I}_r \otimes \hat{a}, \tag{5}$$

$$\widehat{b} = \hat{I}_p \otimes \hat{b} \otimes \hat{I}_p, \tag{6}$$

$$\widehat{c} = \hat{c} \otimes \hat{I}_q \otimes \hat{I}_q, \tag{7}$$

for the mode *a*, *b* and *c*, respectively. The operators $\hat{I}_i\left(i = r, p, q\right)$ is in the form of an identity matrix with *r, p, q* dimensions for mode *a*, *b* and *c*.

If the dissipation processes are ignored, time-evolution of the tripartite quantum system can be expressed by wave-functions and written in the form of number photon states as:

$$\left|\psi(t)\right\rangle = \sum_{r,p,q=0}^{\infty} c_{rpq}\left|r\right\rangle_a \left|p\right\rangle_b \left|q\right\rangle_c \tag{8}$$

where the complex probability amplitudes c_{rpq} corresponds to the *r*-, *p*- and *q*- photon Fock states in mode *a*, *b* and *c*, correspondingly.

The evolution of our system can be regulated by a unitary operator defined from total Hamiltonian as [9]

$$\hat{U} = \exp\left(-i\hat{H}t\right). \tag{9}$$

Supposing that all interactions here are weak compared to nonlinearity constants, we can explain that the transition of the state can behave as a resonant case [14], [15]. The system's evolution is confined to the limited resonant transition states corresponding to $\{p,q,r\} = \{0,1\}$, whereas the others can be neglected. Applying the operator $\hat{U}$ on the initial state, our generated wave function can be obtained as:

$$\left|\psi(t)\right\rangle = \sum_{r,p,q=0}^{\infty} c_{rpq}\left|r\right\rangle\left|p\right\rangle\left|q\right\rangle = \hat{U}\left|\psi(0)\right\rangle. \tag{10}$$

In this work, we define the dimension of each subspace as equal to ten and assume that $\chi_a = \chi_b = \chi_c = \chi = 1$ are proportional to the third-order susceptibilities, $\chi_{ab} = \chi_{bc} = \chi_{ac} = \tilde{\chi} = 1$, and $\varepsilon_{ab} = \varepsilon_{bc} = \varepsilon$ are real number.

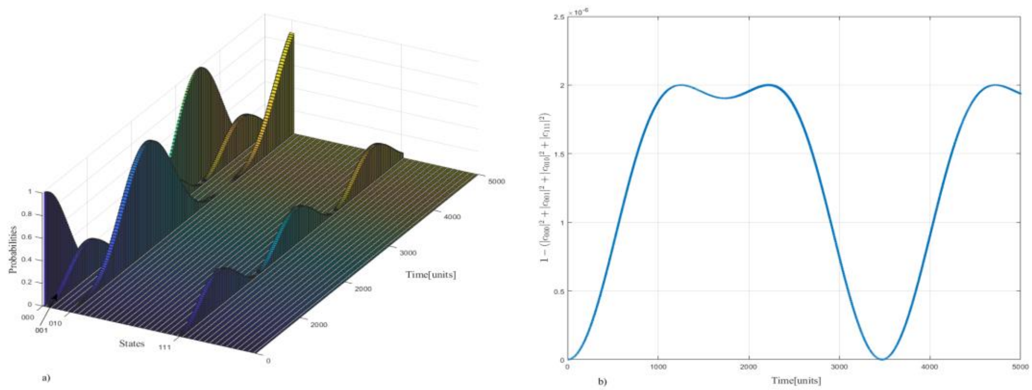

***Figure 1**. a) Time evolution of probabilities of the resonant states $\left|0\right\rangle_a\left|0\right\rangle_b\left|0\right\rangle_c, \left|0\right\rangle_a\left|0\right\rangle_b\left|1\right\rangle_c$, $\left|0\right\rangle_a\left|1\right\rangle_b\left|0\right\rangle_c$, and $\left|1\right\rangle_a\left|0\right\rangle_b\left|0\right\rangle_c$. b) Deviation from the unity of the sum of probabilities. The parameters are $\chi = 1$, $\tilde{\chi} = 1$, $\alpha = \gamma = 10^{-3}$, $\varepsilon = 0{,}8.10^{-3}$.*

As it can be seen from Fig.1a, the time evolution of probabilities of resonant states, when cross-coupling terms are taken into account. Due to effect of crossing coupling terms, the transition of the states evolved into only four states. They are $\left|0\right\rangle_a\left|0\right\rangle_b\left|0\right\rangle_c, \left|0\right\rangle_a\left|0\right\rangle_b\left|1\right\rangle_c$, $\left|0\right\rangle_a\left|1\right\rangle_b\left|0\right\rangle_c$, and $\left|1\right\rangle_a\left|0\right\rangle_b\left|0\right\rangle_c$, which are totally different from the model discussed in [6] with eight states involved in the transition. Fig.1b displays the deviation of the sum of the probabilities, the maximum deviation is $\sim 10^{-6}$. Thus, once again this confirms that the evolution of wave function is closed within four states with a high accuracy. Therefore, it can be seen that when the interacting constants are sufficiently smaller than nonlinearity coefficients, this system can be referred to as nonlinear quantum scissors. From the standpoint of quantum information theory, one can say that the discussed system can be considered as a three-qubit one, due to the fact that only two states (vacuum and one-photon state) for each of the modes are involved in the system's evolution. Thus, the wavefunction now can be written tin truncation form as:

$$\left|\psi(t)\right\rangle_{cut} = c_{000}(t)|0\rangle_a|0\rangle_b|0\rangle_c + c_{001}(t)|0\rangle_a|0\rangle_b|1\rangle_c + c_{010}(t)|0\rangle_a|1\rangle_b|0\rangle_c + c_{100}(t)|1\rangle_a|0\rangle_b|0\rangle_c$$

It is proposed that a three mode nonlinear oscillator model may generate bipartite and tripartite entangled states via optical state truncation. As measurement of bipartite entanglement, we apply the bipartite negativity which was introduced by Vidal and Werner [3], and generalized to the higher dimensions by S. Lee [13]. This quantity is an entanglement monotone and it is easy to calculate. This concept is derived from Peres–Horodecki (PPT) criterion [11], and the negativity measures negative eigenvalues of a density matrix after performing the partial transposition. If ρ_{ij} is density matrix of 2-mode system, the negativity can be obtained by the following:

$$N\left(\rho_{ij}\right) = \frac{\left\|\rho_{ij}^{T_i}\right\|_1 - 1}{2} \tag{11}$$

here $\rho_{ij} = \mathrm{Tr}_k\left(\rho_{ijk}\right)$; $N\left(\rho_{ij}\right)$ are known as the bipartite negativity extracting from three mode matrix where the partial transpose is made for the subsystem i. For pure states, the negativity is equal to the concurrence, meanwhile this quantity gives greater values than entropy [10].

In order to distinguish among many types of tripartite entangled state which may be produced in our system, we use a Sabin's classification [2] to calculate full negativity:

$$N_{\rho_{abc}} = \left(N_{a-bc}N_{b-ac}N_{c-ab}\right)^{1/3}, \tag{12}$$

where N_{i-jk} *(i, j, k=a, b, c)* (different from (11)) can be calculated when we see *ij* as a subsystem which is equivalent to subsystem *i*.

To classify tripartite entanglement, Sabin and Garcia [2] have proposed in their report an entanglement catalog for both mix and pure states in six subtypes from fully separable state to maximal one. Type 0-0 for fully separability, type 1-1 for biseparable states, and type 2 is for fully entanglement states. For our system we are interested in type 2 with 4 subtypes as in the table below.

Table 1. *The catalog for tripartite entanglement*

reduced entanglement	type of tripartite entanglement	tripartite entangled states
$N_{ij}=N_{jk}=N_{ik}=0$	*2- 0*	GHZ-type states
$N_{ij}\neq 0;\ N_{jk}=N_{ik}=0$	*2- 1*	
$N_{ij}\neq 0;\ N_{jk}\neq 0;\ N_{ik}=0$	*2- 2*	star shaped-type state
$N_{ij}\neq 0;\ N_{jk}\neq 0;\ N_{ik}\neq 0$	*2- 3*	W -type class

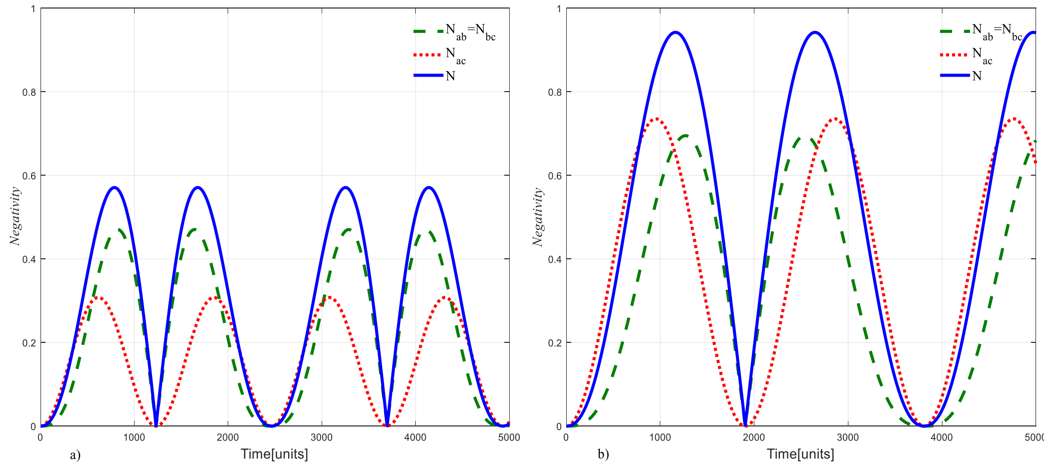

Figure 2. *Time evolution of reduced negativity and tripartite negativity for* a) $\varepsilon = 1,5.10^{-3}$ b) $\varepsilon = 0,5.10^{-3}$ and $\chi = 1$, $\tilde{\chi} = 1$, $\alpha = \gamma = 10^{-3}$.

Fig.2a and 2b show the negativity of bipartite and tripartite system with two different values of interaction strength. From these two figures, we can see that bipartite quantum entanglement is generated not only for the pair a-b, b-c (dashed-line) but also for the pair a-c (doted -line) for both values of interaction strength ε. Thus, the boundary oscillators can generate a bipartite entanglement state through interacting with the central one and depends directly on ε. Thus, by choosing the value of the magnitude of the linear interaction, we can influence the degree of the neighboring bipartite, tripartite - entanglement and time of appearance of the entanglement.

Following the time evolution, it is recognized that the maxima value of tripartite negativity is nearly unity corresponding to the smaller value of interaction (Fig.2b), this means the maximally tripartite entangled state is nearly created. For other time evolution, inseparable state is produced with smaller probabilities and associated with the W-like basic state (Fig.3) which may be expressed as [16].

$$|W_1\rangle = \frac{1}{\sqrt{3}}\left(|0\rangle_a|0\rangle_b|1\rangle_c + |0\rangle_a|1\rangle_b|0\rangle_c + |1\rangle_a|0\rangle_b|0\rangle_c\right), \tag{13}$$

$$|W_2\rangle = \frac{1}{\sqrt{3}}\left(|0\rangle_a|0\rangle_b|1\rangle_c - |0\rangle_a|1\rangle_b|0\rangle_c + |1\rangle_a|0\rangle_b|0\rangle_c\right).$$

When we compare these results with the ones presented in [16] for the system without crossing coupling term, in which the authors stated that the possibility of producing entangled W-states, the dynamics in our system is richer and the model can produce other types of 3-qubit entangled states with high probabilities. Though the W-type state is not a maximal entangled state, this class was confirmed having the highest robustness against the loss of one qubit [14]. The present system can be a source of generation of tripartite entangled states in the form W state, which is a remarkably simple quantum information problem to apply to quantum teleportation and quantum secure communication and so on.

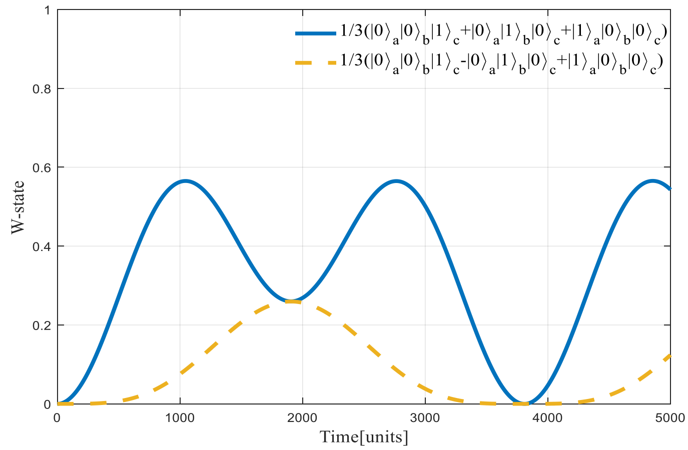

Figure 3. *Time evolution of fidelity corresponding to W-type states for* $\varepsilon = 0,5.10^{-3}$ *and* $\chi = 1$, $\tilde{\chi} = 1$, $\alpha = \gamma = 10^{-3}$.

3. Conclusion

In this work, we have studied the model of a chain of three nonlinear Kerr-type oscillators, with two boundaries that are coupled to the center oscillator by linear interaction and excited by external excitation fields. The system can behave as perfect three-mode nonlinear quantum scissors and can be treated as a 3-qubit system. Both bi- and tripartite entanglement have been discussed. In the case of the 2-qubit entanglement, we can detect not only entanglement between the pairs of oscillators, but also the entanglement can be generated between two oscillators even though they are not directly coupled together. For the case of 3-qubit entanglement, it is possible to obtain W-type class. Thus, the effective Hamiltonian describing our model is not only a potential source of various bi- and tri-partite entangled states, but also stimulating for the discovery of numerous types of quantum correlations as well as relations among them.

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STRATEGIES TO CORRECT ERRORS IN PRONOUNCING FINAL CONSONANT CLUSTERS BY SECOND-YEAR ENGLISH MAJOR STUDENTS AT HONG DUC UNIVERSITY

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Abstract: *This article deals with final consonant clusters errors made by the second-year English-majored students in the Faculty of Foreign Languages, Hong Duc University (HDU). Data were generated through a mid-term oral test with the participation of 30 second-year students. The two data collection instruments are used to record students' talk to find out common errors with final clusters that students actually make and then observe their reading out loud the single words to seek out suitable strategies to overcome those errors. The findings of the study indicate that the majority of the second year students tend to omit one or two elements of the final clusters and some of the students make certain minor substitution and insertion errors. Some useful strategies are also suggested for this typical group of second-year students at HDU to deal with their pronunciation errors.*

Keywords: *Consonants, final clusters, English major, errors.*

1. Introduction

English has become an important demand for educational and job opportunities; however, many Vietnamese speakers do not have intelligible English pronunciation so as to be easily understood in direct communication with foreigners. Learners with serious pronunciation errors will often fail in communication. Therefore, it is very essential to research learners' pronunciation errors, and then to develop suitable strategies to improve articulation.

Among many pronunciation errors that learners of English as a second language are likely to make (i.e. intonation, stress, ending sounds, etc.), errors with final consonant clusters can be considered serious because "learners' inability to produce final consonant clusters can lead to incomprehensibility" [2, p.55]. Along the same line, Celce-Murcia, Brinton & Goodwin [3] supported that inaccurate pronunciation of consonant clusters can make English language learners' speech difficult for native speakers to understand, particularly in cases where the learners use epenthesis to break up clusters or omit a consonant in a cluster [1].

Serious as it might be, problems with final clusters can be considered as one typical pronunciation error of Vietnamese learners. In an article about common challenges faced by Vietnamese learners, Deshayes [5] firmly stated that "English

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consonant clusters give Vietnamese learners problems not only because they do not have these consonant combinations in their own language, but also because they produce a variety of syllable types".

Being an English teacher at Hong Duc University (HDU), I realize that many of my students encounter difficulties in pronouncing English final clusters though they are able to produce single consonants accurately. For the above reasons, I have conducted this research entitled *"Strategies to correct errors in pronouncing final consonant clusters by second-year English major students at HDU".*

2. Some main theoretical terms

2.1. Consonant sounds

According to Peter Roach [11], consonants are "sounds in which there is obstruction to the flow of air as it passes from the larynx to the lips". Specifically, consonant sounds are the sounds in the production of which two articulators come close together so that air stream is obstructed and cannot get out freely.

Consonants can be described in terms of the manner of articulation, the place of articulation and voicing. Kelly [8] and Roach [11] categorized the 24 consonants into 6 groups:

Plosive	Nasal
Fricative	Lateral
Affricative	Approximant

2.2. English final consonants and consonant clusters

Any consonant except h, r, w and j may be a final consonant. When there are two or more consonants at the end of the word (called final cluster), the terms "pre-final" and "post-final" consonants are used. These clusters will be investigated in my study.

Pre- final includes: /m, n, ŋ, l, s/

Post-final includes: /s, z, t, d, θ/

Two consonant clusters:

Pre-final followed by a final consonant

Consonant plus post-final:

E.g.: thi**nk,** importa**nt**, he**lp**, hea**lth**, ca**ts**, etc.

Three consonant clusters:

Pre-final plus final plus post-final (e.g. he**lped**, twe**lfth**, ba**nks**, etc.)

Final plus post-final plus post-final (e.g. te**xt**, fi**fths**, la**psed**, etc.)

Four consonant clusters:

Most are pre-final plus final plus post-final plus post-final.

E.g: pro**mpts**, twe**lfths**.

Occasionally, there is one final and three post-final consonants.

E.g: si**xths**, te**xts**.

2.3. Final consonant clusters errors

According to previous studies, the errors with consonant sounds can be classified into 6 types:

1. Cluster reduction. This is the “deletion of one or more consonants from a target cluster so that only a single consonant occurs at syllable margins” [5; 217].

2. Cluster simplification. The error occurs when one/some elements of a cluster being is/ are produced in a different manner from the target phoneme [5].

E.g. green - pronounced as [gwin]; bread - pronounced as [bwed]

3. Epenthesis. This is the insertion of some vowel (normally a schwa) between cluster elements [6] [12].

E.g. drive /draiv/ pronounced as [dəraiv]

4. Coalescence. It occurs when the yielded pronunciation contains a new consonant composed of features from the original consonants.

E.g. Swim-pronounced as [fim]. It was explained that because the [+fricative] feature of /s/ cooccurs with the [+labial] feature of /w/, resulting in a labial fricative, [f] [6] [12].

5. Omitting nasal and liquid sounds. In consonant clusters consisting of pre-final + final consonants with nasals (/n/, /m/) or liquids (/r/, /l/) as the first element, (/m, n, l, r/ + final consonant), nasals and liquids sounds are often omitted [10].

E.g. went →wet belt → bet

6. Phonetically possible spelling. In representing the first consonant of a cluster, spellers tend to spell words in an inaccurate but phonetically plausible ways [14].

E.g. trap → chap. It was explained that because “ch” closely resembles the sound of the initial blend “tr”. Treiman (1985) explains that this “ch” spelling reflects the release of /t/ in the context [4] [10].

3. The study

3.1. Subjects of the study

Participants in the research are 30 second-year students, who were chosen randomly from the 3 second-year classes of the Faculty of Foreign Languages, HDU. All of them had completed the Pronunciation course offered to first-year students at HDU. Hence, they had got not only basic ideas of pronunciation rules but also a certain awareness of their own pronunciation problems which might have been corrected by teachers.

At the time of the research, these English major subjects, aged from 19 to 21 years old, had worked with the textbook “Achievers B2” by Martin Hobs [10] for speaking skills. The total time allotted to speaking skills for the third term at HDU English majors is 63 class hours. The mid-term oral test occurs in week 8 or week 9 in each class with the same speaking topics.

3.2. Data collection procedure

Data collection was divided into two steps as follows:

Stage 1 (Recording): The data collection was administered through an oral test. This is the mid-term oral test that students had to participate in to get the mid-term marks which make up 20% of the total score in the semester. During the test, each of the students was requested to answer the questions on different topics in about eight to ten minutes. Everything they presented was recorded with the aid of an mp3 recorder which was placed on the table in the test room to get the best audio quality, and was later used for analysis to describe and categorize their errors with final clusters.

Stage 2 (Observation): All words with mispronounced final clusters collected from the recordings were then used for the participants to read out loud. The aim of the stage is to answer the question whether or not the students have the same problems in the test and in their real speech. From this, it is expected to suggest relevant and suitable strategies to correct the errors. Therefore, the speaker himself/ herself would read out loud the errors that he/ she made, paying attention to the final clusters when pronouncing the words. When they read, the researcher took note of any mispronounced final clusters in a checklist.

3.3. Data analysis procedure

The data analysis procedure included two phases:

Phase 1: All information collected from the subjects during the oral test was used for analysis. All the errors made by students were counted in terms of types, frequency and seriousness. The researcher then classified those types of errors and presented them in form of charts and tables. Tape(s) recorded from the oral test were given to a group of three English lecturers at the Faculty of Foreign Languages, HDU for pronunciation evaluation. The evaluators, informed in advance of the purpose of the evaluation, were also given a checklist so that the results would be more precise and suitable for the purpose of the research. After getting the result, they gave a written feedback to the researcher and participated in a discussion to work out the final results. Then, data were processed by using the descriptive statistics, working out the common kinds of mistakes.

Phase 2: The results from recording analysis were used to design a checklist for observation process. The results collected from observation were then compared with the results from recording analysis so as to suggest suitable solutions to students' pronunciation errors with final clusters.

4. Findings and discussions

4.1. Findings from recordings and discussions

The data from recording analysis helped reveal (1) the final clusters errors that the second year students at faculty of Foreign Languages, HDU often make; and (2) which errors are the most common ones in their pronunciation. It should be noted that the most common one was chosen owing to the number of subjects that made the errors and the number of times they appear when students presented the talk.

4.1.1. Overview of the errors with final clusters of 2nd year English-majored students at HDU

From analyzing the data collected from recordings, 230 pronunciation errors related to final clusters were found, including 200 reduction errors and 30 substitution errors. Below is the chart that shows the percentage of the two types of errors:

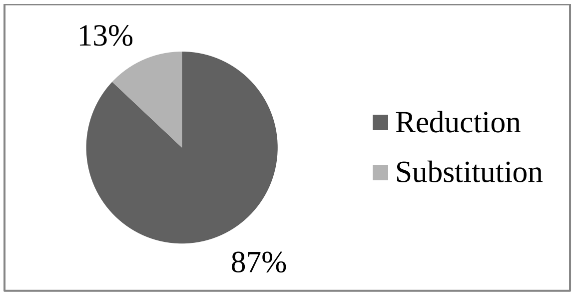

Chart 1. *Types of errors*

As can be seen from the chart, reduction (occured 200 times) is more common than substitution (occurred 30 times). In other words, reduction is the major problem that the second-year students at HDU face.

Basing on the number of students who commit the errors, we can conclude that reduction is the most common error. Of all the subjects, 100% made errors with consonant reduction (30 students) and 60% (18 students) made substitution errors.

Interestingly, no insertion error was found in the data although this error still appeared in some previous research [8] [10].

4.1.2. Details of the errors with final clusters of second-year English major students at HDU

4.1.2.1. Reduction

As mentioned in the previous part, reduction is the most common error that the 2nd year students at HDU make. Below is the table which demonstrates the details of reduction errors found in the data from the recordings.

Table 1. *Reduction errors found in the data from recordings*

Consonant clusters			**Sound(s) omitted**	**Number of subjects with errors**	**Occurrence times**
Pre-final + Final + (Post-final)	Pre-final /l/	/ld/	/l/, /d/ or /ld/	7	8
		/ls/	/s/	1	1
		/lp/, /lps/	/l/	6	7
		/lz/	/z/	1	1
		/lf/	/f/	1	1
		/lθ/	/θ/	5	5
		/lt/	/t/	4	4
		/lvz/	/v/	1	1
		/lvd/	/vd/	1	1
		/lm/	/l/	1	1

	Pre-final /n/	/nt/ or /nts/	/t/ or /ts/	23	39
		/nd/ or /ndz/	/d/ or /dz/	22	43
		/ns/	/s/	1	1
		/nz/	/z/	2	2
		/ndʒ/, /ndʒd/	/dʒ/ or /dʒd/	2	2
		/nθs/	/θ/	1	1
	Pre-final /m/	/mz/	/z/	1	1
		/mp/	/p/	1	1
	Pre-final /ŋ/	/ŋk/	/k/	3	3
	Pre-final /s/	/st/	/t/	22	48
Final + Post-final(s)		/ts/	/s/	1	1
		/pts/	/t/	1	1
		/kt/ or /kts/	/t/ or /ts/	9	11
		/ks/ or /kst/	/s/ or /st/	5	6
		/vd/ or /vz/	/d/ or/z/	4	4
		/bz/	/z/	1	1
		/gd/	/d/	1	1
		/tʃt/	/t/	1	1
		/fts/	/s/	1	1
		/dz/	/z/ or /zd/	2	2

From the Table 1, it can be seen that the errors with final clusters /nt(s), nd(z), st/ are very common among the subjects. The frequency these final clusters appeared in the subjects' talk is also relatively high (/nt(s)/ - 39 times; /nd(z)/ - 43 times; /st/- 48 times). Therefore, it should be noted for correcting. In addition, some of the sounds such as /θ, dʒ/ at the end of words are really hard for Vietnamese learners to pronounce, so subjects tend to delete them. Details of the two types of final clusters are as followed.

Considering the first case (Pre-final + Final + (Post-final)), there were two tendencies to which the final clusters are mispronounced basing on the pre-final consonants: deleting the pre-final and deleting the final or post-final sounds.

Firstly, when the liquid sound /l/ stands as the pre-final, it tends to be omitted. This type of error was made by 33% of the subjects (10 students).

E.g. child /tʃaild/ → /tʃaid/
help /help/ → /hep/
film /film/ → /fim/

The consonant following /l/ is often deleted (accounting for 53%).

E.g. difficult /difikəlt/ → /difikəl/
health /helθ/ → /hel/
else /els/ → /el/

Secondly, when /n/, /m/, /ŋ/ - nasal sounds – stand as pre-final, the students tend to delete all or keep the first and the last sound in a cluster while deleting the middle ones. The phenomenon was found with 100% of the subjects.

E.g. went /went/ → /wen/
environment /invairənmənt/ → /invairənmən/
friends /frendz/ → /fren/

Thirdly, when the pre-final sound is a fricative /s/, the students tend to delete the final sounds. 26 students made this error (87%).

E.g. fast /fa:st/ → /fa:s/
first /fɜːst/ → /fɜːs/
ask /aːsk/ → /aːs/

Considering the second case {Final + post-final(s)}, it is noted that the final consonant is hardly deleted while the second element of two-element clusters and third of three-element clusters are often omitted. Many students just pronounced the first consonants of the long clusters and deleted all the consonants that go after them.

E.g. next /nekst/ → /nek/ (/s/ & /t/ were deleted)
mixed /mɪkst/ → /mik/

There were a few cases of deleting a second element of three-element clusters. For example, accepts /əksepts/ was pronounced as /əkseps/ (/t/ was deleted).

In short, the analysis above shows that the students made the sound omission error. It is easy to understand why final clusters were omitted so frequently. As known, Vietnamese is monosyllabic language, so the students never have to pronounce clusters of consonants. What's more, the habit of "swallowing" the ending sound in the mother tongue is in fact a negative transfer that inhibits the pronunciation of ending sounds in general and final clusters in particular in the target language.

4.1.2.2. Substitution

Of the two common final cluster errors that the subjects committed**,** the substitution errors come second. The following table incorporates the data on students' substitution errors found from recordings.

Table 2. *Substitution errors found in the data from recordings*

Consonant clusters	Sound(s) substituted	No. of subjects with errors	Repetition times
/lθ/	θ=t or 'th' in Vietnamese	2	2
/ʃt, st/	ʃ=s	2	2
/pt/	t = d	1	1
/nz, mz, dz, vz, lz, ŋz/	z = s	14	21
/ndʒ/	dʒ = z	2	2
/nt/	t = s	1	1
/nd/	d =t	1	1

As has been shown in Table 2, the subjects tend to replace the English sound by the Vietnamese one or confuse similar sounds. The English sound replaced by a Vietnamese one is /θ/, for example, health /helθ/ → θ pronounced as 'th' in Vietnamese. It can be explained that this sound is strange to Vietnamese speakers. Because of the influence of their mother tongue, the students simply substitute them with a similar Vietnamese sound.

Regarding sound confusion, the most frequent errors are /s/ and /z/ (repeated 21 times). For instance, loves /lʌvz/ → /lʌvs/; kids /kidz/ → /kids/. The mispronunciation of /z/ to /s/ sounds may be due to the fact that the students often push the air through the mouth too hard.

Also in reference to the confusion of sounds, the mispronunciation of /dʒ/ to /z/ may be due to the carelessness and laziness of the students. The students who made this kind of mistakes usually do not try to find out how the tongue acts in each case, instead they produce all these sounds similarly which results in their mispronunciation as found in this study. The mispronunciation of /θ/ to /t/ may be because of the difficulty the students have when articulating the sound /θ/ at the end of the word.

4.2. Findings from observation and discussions

The data collected from taking notes of each student's reading out loud the problematic words found from recordings are shown in the following table:

Types of errors	Times
Reduction	34
Substitution	7
Insertion	6

Table 3. *Errors from reading out loud the 230 problematic words from recordings*

The data from observation show that the subjects still made mistakes when they read single words. The most common error that the subjects made is reduction error, and it should be noted that there appear insertion errors.

Details of errors from observation are as follows:

Table 4. *Details of errors from observation data*

<table>
<tr><th>Types of errors</th><th>Consonant clusters</th><th>Notes</th></tr>
<tr><td rowspan="2">Reduction</td><td>Two-consonant clusters:
/lθ, lm, lp, lz/
/lt/
/dz/
/ndʒ/
/nd/</td><td>
/l/
/lt/ or /t/ → deleted
/d/
/dʒ/
/n/</td></tr>
<tr><td>Three-consonant clusters:
/nts, pts, kst, ndz, kts, nθs, lvz, lvd, fts/.
/lps/</td><td>The middle sound of a three-consonant cluster is deleted.
/l/ is omitted.</td></tr>
<tr><td>Insertion</td><td>/ld, pt/</td><td>/ə/ is inserted into the middle of a cluster.</td></tr>
<tr><td>Substitution</td><td>/lθ, ʃt, ndʒ, dz/</td><td>θ = t, ʃ = s, dʒ =z, z=s</td></tr>
</table>

As can be seen from Table 4, the subjects mainly made errors with long clusters and clusters with difficult sounds. The reason is perhaps that students had little time to practise them in the Pronunciation course mentioned above. Also, the teacher might not have raised their students' awareness enough about these clusters in particular and clusters in general.

4.4. Strategies to correct common errors in pronouncing final clusters

It is stated in the previous discussion that the difficulty with final clusters may result from teachers' neglect, students' carelessness or laziness, and the negative influence of mother tongue. Within the limited scope of this article, I would like to suggest some activities as well as techniques for correcting the final clusters errors that the thirty second-year students actually made.

Firstly, the results show that students tend to make more final clusters errors in spontaneous speeches than when reading single words. Therefore, it is important to help them form a habit for pronouncing these final clusters. It is impossible to form a habit without practice. The following activities have been compiled and adapted from Celce-Murcia, Brinton & Goodwin [3], Pham Thi Cam Chi [4], and Deshayes [5] in order to help students practise final clusters.

Brainstorming: Ask students to think of words that contain the target sound of the lesson. When students provide enough words, give them communicative activities so that they can practice the sound using those words.

The following example is a brainstorming task to practise the final cluster /nt(s)/:

Ask students to find at least five words containing the final cluster /nt(s)/.

E.g.: went, plant, excellent, want, important, parents, restaurants, spent

Follow-up activity: Work in pairs. Tell your partner what you did in your last summer holiday using at least five words that you have just listed above.

E.g: Last summer, I <u>went</u> to Hue with my <u>parents</u>. We were there for three days. We <u>spent</u> most of the time sightseeing there. We visited a lot of places such as Thien Mu pagoda, Khai Dinh mausoleum, and many other mausoleums. Also, we went along Huong River by boat. Huong River in the evening was very romantic. The local food, especially "Bun bo Hue", "Che Hue" were <u>excellent</u> there. I had a good time there, and I really <u>want</u> to go back to Hue soon.

Dialogues: With a word list containing the target sound of the lesson, teachers can ask students to work in pairs and create their own dialogues using those words. Next, students practise the dialogues they have created.

E.g.: A brief dialogue might be:

Ted: I couldn't finish the *sixth* problem.

Joe: That's because you forgot to reduce 6/6/ (*six/sixths*) to 1.

Short oral presentation: Teachers ask students to find at least five English words with final clusters on a certain topic. Then each learner presents a personal list to the class and makes a short oral presentation that includes at least five of the words. Classmates should evaluate the speaker's production of consonant clusters as to how accurate, natural, and easily intelligible they sound.

Secondly, the results indicate that the subjects in this research had difficulties pronouncing long clusters (three-element clusters), clusters containing the complex sound such as /ʃt, ndʒ/, and clusters with the consonant "l" and its following consonants. To help correct these final clusters errors, I adopted some teaching suggestions from other researchers [2] [3] [5] [8] [9]. Hopefully, these suggestions can partly lessen the students' problems.

Practicing using two words: For example, to practice the final cluster /ld/ as in "field", use the phrase "feel down". The students can gradually eliminate more and more of the second word. E.g: Feel down → feel dow → feel d → field.

Breaking down consonant clusters: Add and change sounds gradually to practice long clusters, for instance, 'six', 'sixth', sixths'. Practice slowly at first and then speed up as confidence increases.

Some important deletions made by native speakers of English should be noted for students as follows: (1) the loss of a fricative when two or more fricatives occur together; for example, /θ/ is lost in *asthma*, /ð/ is lost in *clothes brush*; (2) the deletion of /t/ and /d/ in informal speech when they occur between two other consonants (e.g. frien**d**s, bes**t** man, chil**d**'s); (3) the loss of /k/ in similar contexts, e.g. asked.

A sample dialogue can be used for students' practising cluster simplification strategies as follows:

Vet: What seems to be the problem with Peppy?

Pet owner: Well, he just isn't very peppy, Doc. He <u>acts</u> so tired all the time. He just <u>lifts</u> his head up and sighs.

Vet: And this started two <u>months</u> ago? Can you give me some more <u>facts</u>?

Pet owner: Sure. One of Peppy's big <u>strengths</u> as a guard dog is his <u>bursts</u> of energy. I <u>asked</u> him to fetch the newspaper yesterday and he left three-<u>fourths</u> of it on the doorstep. What does your medical textbook say about that?

Vet: Well, let me look it up under "listless dogs." It says here that "four/<u>fifths</u> of all listlessness in dogs is due to poor diet." Why don't I give you some pep pills? Feed him one every day and we'll see how he <u>acts</u> next week.

5. Conclusion

There are three major types of errors that the second year students at HDU often make with final clusters: reduction, substitution and insertion. The first type of error, reduction, is committed by most of the second-year students. For the second type of error, substitution, students tend to replace an English sound by a Vietnamese one or confuse similar sounds. The results show that students often confuse /z/ with /s/, /ʃ/ with /s/, or /dʒ/

with /z/. The last type - Insertion - which does not appear in students' real speech but in their reading out loud single words- is also one error with final clusters that students make.

Some strategies are suggested for students, including brainstorming, dialogues, short oral presentation, information gap activity, Practicing using two words and Breaking down consonant clusters. Those strategies focus on developing students' habit of pronouncing final clusters in the speaking process as well as helping them overcome the difficult final clusters.

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SOME FIXED POINT THEOREMS FOR FISHER-TYPE CONTRACTIVE MAPPINGS

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Abstract: *In this paper, we provide some new fixed point theorems for mappings satisfying Fisher - type contractive conditions in metric spaces. Some examples are also given to illustrate our results.*

Keywords: *Boundedly compact, fixed point property, metric space, T-orbitally compact space.*

1. Itroduction and Preliminaries

In 1976, Fisher [1] introduced and proved several results for mappings satisfying different contractive conditions. One of them is the following fixed point theorem.

Theorem 1.1. ([1]) *If T is a continuous mapping of the compact metric space X into itself satisfying the inequality*

$$\left[d\left(Tx,Ty\right)\right]^2 < d\left(x,Tx\right).d\left(y,Ty\right)+c.d\left(x,Ty\right)d\left(y,Tx\right) \tag{1.1}$$

for all distinct x, y in X, where $0 \le c$, then T is a fixed point mapping. Futher, if $0 \le c \le 1$, then the fixed point of T is unique.

In this article, we call mappings satisfying the condition (1.1) as Fisher-type contractive mappings. It is well-known that every Banach contraction mapping is continuous. Here, Fisher also requires the continuity of the mapping. We consider the relationship between the continuity of mappings and the existence of fixed points, as well as the necessity of the compactness of the underlying spaces.

First of all, we recall some definitions that will be used in this article.

Definition 1.1. A metric space (X,d) is said to be boundedly compact if every bounded sequence in X has a convergent subsequence.

Definition 1.2. Let (X,d) be a metric space and T be a self-mapping on X. The orbit of T at $x \in X$ is defined as: $O_x(T) = \left\{x, Tx, T^2x, T^3x, \ldots\right\}$

Definition 1.3. Let (X,d) be a metric space and T be a self-mapping on X. Then, X is said to be T-orbitally compact if every sequence in $O_x(T)$ has a convergent subsequence for all $x \in X$.

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In ([2]), H. Garai et al. have shown that T-orbital compactness of a space depends on the mapping T defined on it.

Definition 1.4. Let (X,d) be a metric space and $T:X\to X$ be a self mapping. T is said to be orbitally continuous at a point z in X if for any sequence $\{x_n\}\subseteq O_x(T)$ for some $x\in X$, $x_n\to z$ as $n\to\infty$ implies $Tx_n\to Tz$ as $n\to\infty$.

Definition 1.5. Let (X,d) be a metric space and $T:X\to X$ be a mapping. For $x_0\in X$ the sequence $\{x_n\}$ defined by $x_n=Tx_{n-1}$ for $n\geq 1$ is called Picard iteration sequence with the initial point x_0. The mapping T is said to be a Picard operator if it has a unique fixed point and every Picard iteration in X converges to the fixed point.

2. Main results

Now, we are in a position to state our main results.

Theorem 2.1. *Let (X,d) be a compact metric space and let $T:X\to X$ such that*

$$\left[d\left(Tx,Ty\right)\right]^2<d\left(x,Tx\right).d\left(y,Ty\right)+c.d\left(x,Ty\right)d\left(y,Tx\right)$$

for all $x,y\in X, x\neq y$ and $0\leq c\leq 1$. Then, T has a unique fixed point.

Proof. Set $m=\inf\{d(x,Tx):x\in X\}$,

Then, there exists a sequence $(x_n)\subset X$ such that $\lim\limits_{n\to\infty}d\left(x_n,Tx_n\right)=m,$ and, by the compactness of X, there exists a subsequence (x_{n_k}) of (x_n) such that $x_{n_k}\to u\in X$, $Tx_{n_k}\to w\in X$ as $k\to\infty.$ We have $d\left(x_{n_k},Tx_{n_k}\right)\to d\left(u,w\right)=m$ as $k\to\infty.$

If $m>0$ then $u\neq w$.

If $Tx_n=Tw$ for *n* in an infinite subset *I* of $\mathbb{N}$, then $Tx_n=Tw\to w$. So w is a fixed point.

When I is a finite set, we denote $(x_n')=(x_n)\setminus\{x_m:Tx_m=Tw,m\in I\}$.

Assume now that $Tx_n\neq Tw\ \forall n$, we have

$$\left[d\left(Tx_n,Tw\right)\right]^2<d\left(x_n,Tx_n\right)d\left(w,Tw\right)+cd\left(x_n,Tw\right)d\left(w,Tx_n\right),$$

letting $n\to\infty$, we get

$$\left[d\left(w,Tw\right)\right]^2\leq d\left(u,w\right)d\left(w,Tw\right).$$

This implies that

$$d\left(w,Tw\right)\leq d\left(u,w\right)=m.$$

And from

$$\left[d\left(Tw,T^2w\right)\right]^2<d\left(w,Tw\right)d\left(Tw,T^2w\right)+cd\left(w,T^2w\right)d\left(Tw,Tw\right),$$

We obtain $d\left(Tw,T^2w\right)<d\left(w,Tw\right)\le m,$ which is a contradiction. Thus, we must have $m=0$. Therefore, $u=w$.

If $Tx_n\ne Tu$, we have

$$\left[d\left(Tx_n,Tu\right)\right]^2<d\left(x_n,Tx_n\right)d\left(u,Tu\right)+cd\left(x_n,Tu\right)d\left(u,Tx_n\right),$$

Taking the limit as $n\to\infty$, we obtain $\left[d\left(u,Tu\right)\right]^2\le 0.$

This implies that $u=Tu$, i.e., u is a fixed point of T.

Next, we check the uniqueness of u. Arguing by contradiction, let u' be another fixed point of T, then

$$\left[d\left(u,u'\right)\right]^2=\left[d\left(Tu,Tu'\right)\right]^2<d\left(u,Tu\right)d\left(u',Tu'\right)+cd\left(u,Tu'\right)d\left(u',Tu\right)=c\left[d\left(u,u'\right)\right]^2,$$

which is a contradition. The fixed point must therefore be unique. This completes the proof of the theorem.

In Theorem 2.1, the existence of the fixed point of Fisher-type contractive mappings without assuming the continuity of the mapping. However, at fixed point, the mapping is still continuous at the fixed point.

Indeed, let (x_n) be a sequence in X. If $x_n\ne u$, we have

$\left[d\left(Tx_n,Tu\right)\right]^2<d\left(x_n,Tx_n\right)d\left(u,Tu\right)+cd\left(x_n,Tu\right)d\left(u,Tx_n\right)$, which implies that

$$d\left(Tx_n,Tu\right)<cd\left(x_n,u\right). \tag{2.1}$$

By (2.1), if $x_n\to u$ as $n\to\infty$ then $Tx_n\to Tu$ as $n\to\infty$. Thus, T is continuous at the fixed point u.

Theorem 2.2. *Let (X,d) be a boundedly compact metric space and $T:X\to X$ be an orbitally continuous mapping such that*

$$\left[d\left(Tx,Ty\right)\right]^2<d\left(x,Tx\right)d\left(y,Ty\right)+c.d\left(x,Ty\right).d\left(y,Tx\right)$$

for all $x,y\in X$ with $x\ne y$ and $0\le c<1$. Then T is a Picard operator.

Proof. Let $x_0\in X$ be arbitrary but fixed and consider the iterative sequence (x_n), where $x_n=T^nx_0$ for each $n\in\mathbb{N}$. If the sequence (x_n) has two equal consecutive terms, then T must have a fixed point. So, we may assume that no two consecutive terms of (x_n) are equal.

We denote $s_n=d(x_n,x_{n+1})$. Then $s_n>0$ for each $n\in\mathbb{N}$. We have

$$\begin{aligned}
s_n^2&=\left[d\left(T^nx_0,T^{n+1}x_0\right)\right]^2\\
&<d\left(T^{n-1}x_0,T^nx_0\right)d\left(T^nx_0,T^{n+1}x_0\right)+c.d\left(T^{n-1}x_0,T^{n+1}x_0\right)d\left(T^nx_0,T^nx_0\right)\\
&=d\left(T^{n-1}x_0,T^nx_0\right)d\left(T^nx_0,T^{n+1}x_0\right)\\
&=s_{n-1}s_n.
\end{aligned}$$

Thus $s_n < s_{n-1}$. This shows that (s_n) is a strictly decreasing sequence of positive real numbers. Hence, it converges to some $b \geq 0$. For each $n \in \mathbb{N}$, we have

$$s_n < s_{n-1} < \ldots < s_1 = K \text{ (say)}.$$

Thus, for all n, m, one has

$$\begin{aligned}
t^2 &= \left[d\left(x_n, x_m\right)\right]^2 = \left[d\left(T^n x_0, T^m x_0\right)\right]^2 \\
&< d\left(T^{n-1}x_0, T^n x_0\right) d\left(T^{m-1}x_0, T^m x_0\right) + c.d\left(T^{n-1}x_0, T^m x_0\right) d\left(T^{m-1}x_0, T^n x_0\right) \\
&< d\left(T^{n-1}x_0, T^n x_0\right) d\left(T^{m-1}x_0, T^m x_0\right) \\
&+ c\left[d\left(T^{m-1}x_0, T^m x_0\right) + d\left(T^m x_0, T^n x_0\right)\right]\left[d\left(T^{n-1}x_0, T^n x_0\right) + d\left(T^n x_0, T^m x_0\right)\right] \\
&= s_{n-1}s_{m-1} + c\left(s_{n-1} + t\right)\left(s_{m-1} + t\right) \\
&< K^2 + c\left(K + t\right)^2.
\end{aligned}$$

This implies that $(1-c)t^2 - 2cKt - K^2(1+c) < 0,$ or, $t = d\left(x_n, x_m\right) < K\dfrac{1+c}{1-c}.$

Therefore, (x_n) is a bounded sequence in X. By the bounded compactness property of X, (x_n) must have a convergent subsequence, say (x_{n_k}), which converges to some $z \in X$. By the orbital continuity of T, (Tx_{n_k}) converges to Tz. We have

$$\begin{aligned}
s_{n_k} &= d(x_{n_k}, Tx_{n_k}), \\
s_{n_k+1} &= d(Tx_{n_k}, T^2 x_{n_k}).
\end{aligned}$$

Taking the limit as $k \to \infty$, we obtain $b = d(z, Tz) = d(Tz, T^2 z)$.

We are going to show that $b = 0$.

Assume that $b > 0$. Then, $z \neq Tz$ and

$$\left[d\left(Tz, T^2 z\right)\right]^2 < d\left(z, Tz\right) d\left(Tz, T^2 z\right) + c.d\left(z, T^2 z\right).d\left(Tz, Tz\right).$$

Thus, $d(Tz, T^2 z) < d(z, Tz),$ which is a contradiction.

So the sequence (s_n) must converge to 0. For all n, m,

$$\begin{aligned}
\left[d\left(x_n, x_m\right)\right]^2 &= \left[d\left(T^n x_0, T^m x_0\right)\right]^2 \\
&< d\left(T^{n-1}x_0, T^n x_0\right) d\left(T^{m-1}x_0, T^m x_0\right) + c.d\left(T^{n-1}x_0, T^m x_0\right) d\left(T^{m-1}x_0, T^n x_0\right) \\
&< d\left(T^{n-1}x_0, T^n x_0\right) d\left(T^{m-1}x_0, T^m x_0\right) \\
&+ c\left[d\left(T^{m-1}x_0, T^m x_0\right) + d\left(T^m x_0, T^n x_0\right)\right]\left[d\left(T^{n-1}x_0, T^n x_0\right) + d\left(T^n x_0, T^m x_0\right)\right] \\
&= s_{n-1}s_{m-1} + c\left(s_{n-1} + d\left(x_n, x_m\right)\right)\left(s_{m-1} + d\left(x_n, x_m\right)\right).
\end{aligned}$$

This implies that $d\left(x_n, x_m\right) \to 0,$ as $m, n \to \infty.$

Thus, (x_n) is a Cauchy sequence. As the subsequence (x_{n_k}) of (x_n) converges to z, the limit of (x_n) must be z and $z = w$. Assume that $Tx_n \neq Tz$, we have

$$\left[d\left(Tx_n, Tz\right)\right]^2 < d\left(x_n, Tx_n\right) d\left(z, Tz\right) + cd\left(x_n, Tz\right) d\left(z, Tx_n\right).$$

Letting $n \to \infty$, we get $\left[d\left(z, Tz\right)\right]^2 \leq 0.$

This implies that $z = Tz$, i.e., z is a fixed point of T.

Next, we check the uniqueness of z. Arguing by contradiction, assume z' is another fixed point of T. Then

$$\left[d(z, z')\right]^2 = \left[d(Tz, Tz')\right]^2 < d(z, Tz) d(z'Tz') + cd(z, Tz') d(z', Tz) = c\left[d(z, z')\right]^2.$$

Since $0 \leq c < 1$, we obtain $\left[d(z, z')\right]^2 = 0$. This implies $z = z'$.

Therefore, z is the unique fixed point of T. Since we take x_0 as an arbitrary point, for every $x \in X$, the iterative sequence $(T^n x)$ converges to z, i.e., T is a Picard operator.

Example 2.1. Let $X = [1.8, \infty)$ with the usual metric $d(x, y) = |x - y|$ for all $x, y \in X$. Then, (X, d) is a compact metric space. We consider the mapping $T : X \to X$ defined by

$$Tx = \begin{cases} \dfrac{3}{4}x + \dfrac{1}{2} & \text{if} \quad 1.8 \leq x < 2, \\ 2 & \text{if} \quad x \geq 2. \end{cases}$$

Then T satisfies the Fisher -type contractive condition (1.1) with $c = 0.99$.

Proof.

Cases 1: If $x, y \in [1.8, \infty), x \neq y$, then $d(Tx, Ty) = 0$ and the inequality (1.1) holds.

Cases 2: If $x, y \in [1.8, 2), x \neq y$. Let $S = x + y, P = xy$. We have: $S^2 \geq 4P.$

Then $[d(Tx, Ty)]^2 = \dfrac{9}{16}(x - y)^2 = \dfrac{9}{16}(S^2 - 4P),$ and

$$\begin{aligned} d(x, Tx)d(y, Ty) + cd(x, Ty)d(y, Tx) &= \left(\frac{1}{2} - \frac{1}{4}x\right)\left(\frac{1}{2} - \frac{1}{4}y\right) + c\left|\left(x - \frac{3}{4}y - \frac{1}{2}\right)\left(y - \frac{3}{4}x - \frac{1}{2}\right)\right| \\ &= \frac{1}{16}(2 - x)(2 - y) + \frac{c}{16}\left|(4x - 3y - 2)(4y - 3x - 2)\right| \\ &= \frac{1}{16}(4 - 2S + P) + \frac{c}{16}\left|12S^2 - 49P + 2S - 4\right|. \end{aligned}$$

Thus, $d(x, Tx)d(y, Ty) + cd(x, Ty)d(y, Tx) > [d(Tx, Ty)]^2$ which is equivalent to

$$\left(4 - 2S + P\right) + c\left|12S^2 - 49P + 2S - 4\right| > 9\left(S^2 - 4P\right), \text{ or}$$

$$c\left|12S^2-49P+2S-4\right|>9S^2-37P+2S-4 \tag{2.2}$$

If $9S^2-37P+2S-4<0$ then (2.2) holds.

If $9S^2-37P+2S-4\geq 0$ and $c=0.99$, we have

$$0.99\left(12S^2-49P+2S-4\right)=\left(9S^2-37P+2S-4\right)+\left(2.88S^2-11.51P-0.02S+0.04\right).$$

We have

$$\begin{aligned}2.88S^2-11.51P-0.02S+0.04&>2.88\times 4P-11.51P-0.02S+0.04\\&=0.01P-0.02S+0.04>0.\end{aligned}$$

Thus, $0.99\left(12S^2-49P+2S-4\right)>9S^2-37P+2S-4\geq 0.$

This implies that $0.99\left|12S^2-49P+2S-4\right|>9S^2-37P+2S-4,$ then (2.2) holds.

Cases 3: If $x\in[1.8,2), y\geq 2$,

then $[d(Tx,Ty)]^2=\left(\frac{3}{2}-\frac{3}{4}x\right)^2$ and with $c=0.99>\frac{3}{4}$, we have

$$\begin{aligned}d(x,Tx)d(y,Ty)+cd(x,Ty)d(y,Tx)&=c(2-x)\left(\frac{3}{2}-\frac{3}{4}x\right)\\&>\frac{3}{4}(2-x)\left(\frac{3}{2}-\frac{3}{4}x\right)\\&=\left(\frac{3}{2}-\frac{3}{4}x\right)^2=[d(Tx,Ty)]^2,\end{aligned}$$

and the inequality (1.1) holds. Hence, the inequality (1.1) holds with $c=0.99$.

Clearly, X is boundedly compact and T is a mapping satisfying (1.1). T has a fixed point and 2 is the only fixed point of T.

Theorem 2.3. *Let (X,d) be a T-orbitally compact metric space, where $T:X\to X$ is an orbitally continuous mapping such that*

$$\left[d\left(Tx,Ty\right)\right]^2<d\left(x,Tx\right)d\left(y,Ty\right)+c.d\left(x,Ty\right).d\left(y,Tx\right)$$

for all $x,y\in X$ with $x\neq y$ and $0\leq c<1$. Then, T has a unique fixed point z and for any $x\in X$, the sequence of iterates $\left(T^n x\right)$ converges to z.

Proof. Let $x_0\in X$ be arbitrary but fixed and consider the sequence $\left(x_n\right)$, where $x_n=T^n x_0$ for each $n\in\mathbb{N}$.

Since X is T-orbitally compact, the sequence $\left(x_n\right)$ has a convergent subsequence, say $\left(x_{n_k}\right)$, and let $\left(x_{n_k}\right)$ converging to z in X. By the orbital continuity of T, (Tx_{n_k}) converges to Tz.

Now, proceeding as in Theorem 2.2, we can similarly prove that the sequence $(d(x_n, x_{n+1}))$ converges to 0 and that the sequence (x_n) is a Cauchy sequence and hence $x_n \to z \in X$ as $n \to \infty$. Therefore, z is the unique fixed point of T.

Theorem 2.4. *Let (X, d) be a complete metric space and T be a self-mapping on X such that*

1. $\left[d\left(Tx,Ty\right)\right]^2 < d\left(x,Tx\right)d\left(y,Ty\right)+c.d\left(x,Ty\right).d\left(y,Tx\right)\ (0 \le c < 1)$ *for all* x, y *in* X *with* $x \neq y$,

2. for any $x \in X$ *and for any* $\epsilon > 0$, *there exists* $\delta > 0$ *such that* $d\left(T^i x, T^j x\right) < \epsilon + \delta$ *implies* $d\left(T^{i+1}x, T^{j+1}x\right) \le \epsilon$, *for any* $i; j \in \mathbb{N} \cup \{0\}$.

Then, T *has a unique fixed point* z *and for any* $x \in X$, *the sequence of iterates* $(T_n x)$ *converges to* z.

Proof. Let $x_0 \in X$ be arbitrary but fixed and consider the sequence (x_n),

where $x_n = T^n x_0$ for each $n \in \mathbb{N}$. Let the sequence (x_n) do not have two equal consecutive terms, i.e., $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}$.

Then, similar to Theorem 2.1, it is not difficult to check that the sequence of real numbers (s_n), where $s_n = d\left(x_n, x_{n+1}\right)$ is a decreasing sequence and also this sequence is bounded below. Thus, this sequence is convergent and let $\lim\limits_{n\to\infty} s_n = b = \inf\{s_n, n \in \mathbb{N}\}$.

Therefore, $b \ge 0$.

Assume that $b > 0$. Then, there exists $\delta > 0$ and $n \in \mathbb{N}$ such that $s_n < b + \delta$.

This implies that $d\left(x_n, x_{n+1}\right) < b + \delta$.

Thus, by the given condition, we have $d\left(x_{n+1}, x_{n+2}\right) \le b$, i.e., $s_{n+1} \le b$, (s_n) is a decreasing sequence, so $s_{n+2} < s_{n+1} \le b$. This leads to a contradiction. Thus, we must have

$$b = \lim_{n\to\infty} d\left(x_n, x_{n+1}\right) = 0.$$

Now for any $n, m \in \mathbb{N}$, we have

$$\begin{aligned}
\left[d\left(x_n, x_m\right)\right]^2 &= \left[d\left(T^n x_0, T^m x_0\right)\right]^2 \\
&< d\left(T^{n-1}x_0, T^n x_0\right) d\left(T^{m-1}x_0, T^m x_0\right) + c.d\left(T^{n-1}x_0, T^m x_0\right) d\left(T^{m-1}x_0, T^n x_0\right) \\
&< d\left(T^{n-1}x_0, T^n x_0\right) d\left(T^{m-1}x_0, T^m x_0\right) \\
&\quad + c\left[d\left(T^{m-1}x_0, T^m x_0\right) + d\left(T^m x_0, T^n x_0\right)\right]\left[d\left(T^{n-1}x_0, T^n x_0\right) + d\left(T^n x_0, T^m x_0\right)\right] \\
&= s_{n-1}s_{m-1} + c\left(s_{n-1} + d\left(x_n, x_m\right)\right)\left(s_{m-1} + d\left(x_n, x_m\right)\right).
\end{aligned}$$

Thus, $d\left(x_n, x_m\right) \to 0$ as $m, n \to \infty$.

This implies that (x_n) is a Cauchy sequence and, we have, $\lim_{n\to\infty} x_n = z$.

Consider $Tx_n \neq Tz$, we have

$$\left[d\left(Tx_n,Tz\right)\right]^2 < d\left(x_n,Tx_n\right)d\left(z,Tz\right)+cd\left(x_n,Tz\right)d\left(z,Tx_n\right),$$

Taking the limit as $n \to \infty$, we obtain

$$\left[d\left(z,Tz\right)\right]^2 \leq 0.$$

This implies that $z = Tz$, i.e., z is a fixed point of T.

Next, we check the uniqueness of z. Arguing by contradiction, let z' be another fixed point of T, then

$$[d(z,z')]^2 = [d(Tz,Tz')]^2 < d(z,Tz)d(z'Tz') + cd(z,Tz')d(z',Tz) = c[d(z,z')]^2.$$

Since $0 \leq c < 1$, we have $[d(z,z')]^2 = 0$.

This gives $z = z'$.

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ELECTRONIC STRUCTURE OF POLYMORPHISMS PHASES OF LEAD-FREE FERROELECTRIC $Bi_{0.5}Na_{0.5}TiO_3$ MATERIALS

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Abstract: *Bismuth-based ferroelectric ceramics are currently under intense investigation for the potential as Pb-free alternatives to lead zirconate titanate-based ($Pb(Zr,Ti)O_3$-based) piezoelectrics. In this work, first-principle calculations were performed for the electronic structures of sodium bismuth titanate ($Bi_{0.5}Na_{0.5}TiO_3$) materials with all possible crystal symmetries, including rhombohedral, monoclinic, tetragonal, and rhombohedral. We expected that our works could help further understand the role of phase transition in lead-free ferroelectric $Bi_{0.5}Na_{0.5}TiO_3$ materials.*

Keywords: *$Bi_{0.5}Na_{0.5}TiO_3$, Lead-free ferroelectric, First principle calculation.*

1. Introduction

Lead-based ferroelectric $Pb(Zr,Ti)O_3$ materials have been given much attention because their piezoelectric, ferroelectric, and dielectric properties are better than lead-free ferroelectric materials. With the rapid industrial development, the amounts of Pb used in electronic devices have increased, seriously affecting the environment and human health. Thus, the performance of eco-friendly lead-free ferroelectric materials, such as $Bi_{0.5}Na_{0.5}TiO_3$, should be improved [1]. Sodium bismuth titanate BNT materials, first fabricated by Smolensky et al. in 1960 [2], are ferroelectric materials with Curie temperature of 320°C, remanent polarization of 38 $\mu C/cm^2$, and coercive field of 73 kV/cm at room temperate [2] [3]. $Bi_{0.5}Na_{0.5}TiO_3$ materials have a low piezoelectric coefficient of ~70-80 pC/N due to their high coercive field [3]. $Bi_{0.5}Na_{0.5}TiO_3$ materials exhibited rhombohedral structure at room temperature and monoclinic symmetry [4] [5]. Pronin et al. [6] obtained that the first phase transition tetragonal-cubic phase occurred at 320°C, whereas the second phase transition temperature of the rhombohedral-tetragonal phase was 540°C as they determined the Curie temperature. The average symmetries of rhombohedral, monoclinic, and cubic BNT structures are *R3c*, *Cc*, *P4bm*, and *Fm-3m* space groups, respectively.

Recently, optical bandgap, diffuse scattering, infrared, and high-pressure Raman spectra of BNT have been extensively studied experimentally [7-10]. The crystal structure of $Bi_{0.5}Na_{0.5}TiO_3$ materials was determined by neutron powder diffraction at 698 K [7]. In the theoretical aspect, first-principle calculations were widely adopted to study the

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structure, band structure, and electronic properties of perovskite ferroelectric materials, including BNT [11]. BNT with a high-temperature cubic phase is a semiconductor, discovered by Xu and Ching [12]. Bujakiewicz-Koronska and Natazon investigated the elastic properties of BNT crystals [13]. Gröting et. al. [14] calculated the phase stability of BNT under pressure using ab-initio supercell calculations, and Niranjan et. al. [15] investigated the dielectric properties and phonon frequencies for the rhombohedral crystal structural by density functional perturbation theory. Most theoretical studies on BNT focused on rhombohedral and cubic phases, while only a few works were conducted on tetragonal and monoclinic phases. Tetragonal and monoclinic phases are transitional between the rhombohedral and cubic ones of BNT, and they are often related to the interplaying between different phases [16]; it would be beneficial to study all the four phases of BNT under the same approach to provide comprehensive perspective. In this work, we have performed first-principle calculations on BNT for all four phases possible.

2. Experiments

In this work, all DFT calculations were performed using the Cambridge Serial Total Energy Package (CASTEP) module in the Materials Studio software. For exchange-correlation energy, we adopted the generalized gradient approximation (GGA) using Perdew - Burke - Ernzerhof (PBE) exchange-correlation functionals [16], which is sufficiently accurate to describe the crystal structures and electronic properties of perovskite compounds [17]. The energy cutoff of the plane-wave basis functions was set to be 500 eV, which yields sufficiently convergence of the total energy differences better than 10^{-6} eV per atom. The Monkhorst-Pack scheme was used to sample the Brillouin zone; and the k-point sampling method [18] was chosen for the reciprocal space integrations over the Brillouin zone with good convergence for the calculated properties, where the k-point mesh of 5 × 5 × 4 is used. The structures were fully relaxed with convenient primary cells. Four BNT phases, including rhombohedral, monoclinic, tetragonal, and cubic, were simulated using *R3c*, *Cc*, *P4bm*, and *Fm-3m* space groups, respectively. The cubic and rhombohedral BNT structures were assumed from the experimental lattice parameters [19].

3. Results and Discussion

3.1. Crystal structures

Fig.1 shows four different crystal structures of BNT. Adopting of an ordered structure facilitates the calculations and helps avoid computational artifacts [20]. The replacement of Na and Bi atoms at A-sites leads to lower, the symmetries of the rhombohedral, monoclinic, tetragonal, and cubic BNT phases to *R3c*, *Cc*, *P4bm*, *Fm-3m*, respectively. At room temperature, the optimized lattice constants for the conventional cell of the rhombohedral phase are a = 5.501 Å and c = 13.496 Å, which are close to the experimental value of 5.488 Å and 13.504 Å [19]. Meanwhile, at a higher temperature, the lattice constants of monoclinic, tetragonal, and cubic phases have been optimized close to the experimental values, respectively. Table 1 presents the optimized lattice constants of the four BNT phases in comparison to the experimental values by G. O. Jones [7], H. Lü [19] and E. Aksel [21] (within 1%-2%).

Table 1. *Optimized calculated and experimental lattice parameters (Å) and atomic coordinates in four BNT phases*

Rhombohedral phase		
	Calc.	*Ref. [19]*
a	5.501	5.488
c	13.496	13.504
α	90°	90°
γ	120°	120°
Na	(0 0 0.259)	(0 0 0.257)
Bi	(0 0 0.783)	(0 0 0.757)
Ti	(0 0 0.008)	(0 0 0)
	(0 0 0.503)	(0 0 0.500)
O	(-0.338 -0.101 0.567)	(-0.336 -0.126 0.577)
	(-0.233 -0.333 0.405)	(-0.207 -0.330 0.410)
Monoclinic phase		
	Calc.	*Ref. [21]*
a	9.555	9.526
b	5.508	5.483
c	5.634	5.507
α	90°	90°
β	125.742°	125.344°
Na	(0.502 0.750 -0.006)	(0.500 0.750 0)
Bi	(-0.003 0.256 0.036)	(0 0.250 0)
Ti	(0.255 0.243 0.744)	(0.270 0.247 0.742)
O	(-0.014 0.180 0.414)	(0.008 0.194 0.493)
	(0.189 0.486 0.901)	(0.177 0.481 0.862)
	(0.262 0.981 0.951)	(0.241 0.996 0.955)
Tetragonal phase		
	Calc.	*Ref. [7]*
a	5.410	5.519
c	7.925	7.817
α	90°	90°
Na	(0 0.5 0.253)	(0 0.5 0.261)
Bi	(0 0.5 0.759)	(0 0.5 0.761)
Ti	(0 0 0.019)	(0 0 0)
O	(0 0 0.259)	(0 0 0.257)
	(0 0 0.761)	(0 0 0.757)
Cubic phase		
	Calc.	*Ref. [19]*
a	7.765	7.827
α	90°	90°
Na	(0 0 0)	(0 0 0)
Bi	(0.5 0.5 0.5)	(0.5 0.5 0.5)
Ti	(0.25 0.25 0.25)	(0.25 0.25 0.25)
O	(0.25 0.25 0)	(0.25 0.25 0)

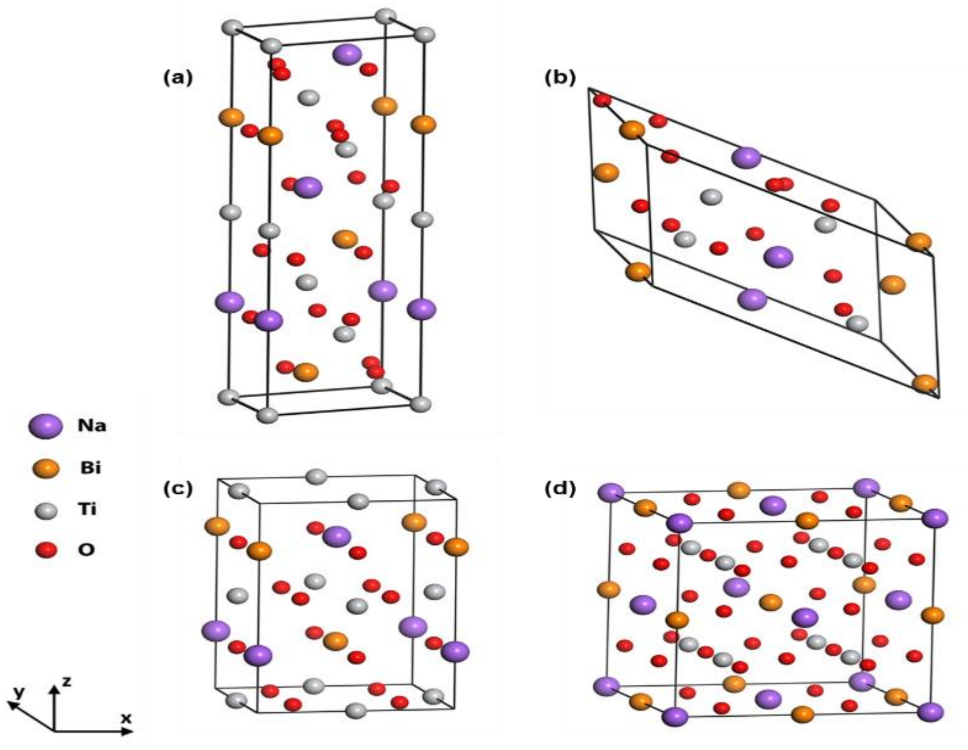

Figure 1. *Crystal structures of (a) rhombohedral, (b) monoclinic, (c) tetragonal, and (d) cubic phase of BNT*

The corresponding atomic coordinates are also given for the four BNT phases in Table 1. Atoms in rhombohedral and monoclinic phases are imposed by the symmetries of space groups *R3c* and *Cc*, respectively, where the symmetries allow them to be relaxed separately. In contrast, atoms in the tetragonal and cubic phases of BNT have higher symmetry structures. Ti atom of the cubic phase has six nearest O atoms, and Na or Bi atom has twelve nearest O atoms. Thus, Ti atoms are in the center of oxygen-octahedral and Na, or Bi is in the center of oxygen-cuboctahedral [19]. From the cubic phase, the displacements of Na, Bi, and Ti atoms with respect to the center of the O cages are calculated for the tetragonal phase. In addition, Na and Bi atoms in the rhombohedral and monoclinic phases have been arranged into layer-by-layer, being favorable conditions for supercell calculations.

3.2. Electronic structures

There is some disparity between different calculations on the band structure of BNT. For the rhombohedral phase, H. Lü [19] gave an indirect bandgap of 2.82 eV, while R. Bujakiewicz-Koronska [13] predicted a value of about 2 eV. For the tetragonal and cubic phases, H. Lü [19] gave a bandgap of 2.29 eV and 1.96 eV, respectively. As this work results are shown in Fig.2, the rhombohedral phase has a direct bandgap of 2.764 eV, while the monoclinic phase, tetragonal phase, and cubic phase have indirect bandgaps of 2.575

eV, 2.189 eV, and 1.473 eV, respectively. For the rhombohedral, monoclinic, and cubic phases, the top of the valence band is located at the G(0 0 0) point. The bottom of the conduction band for the rhombohedral phase is also located at the G point, while for the monoclinic and cubic phases, it is at the B and R points, respectively. The direct bandgaps were estimated at 2.764 eV, 2.580 eV, 2.192 eV, and 2.150 eV for rhombohedral, monoclinic, tetragonal, and cubic BNT materials, respectively. Overall, the bandgap values of the four BNT phases indicate that they are suitable optical materials.

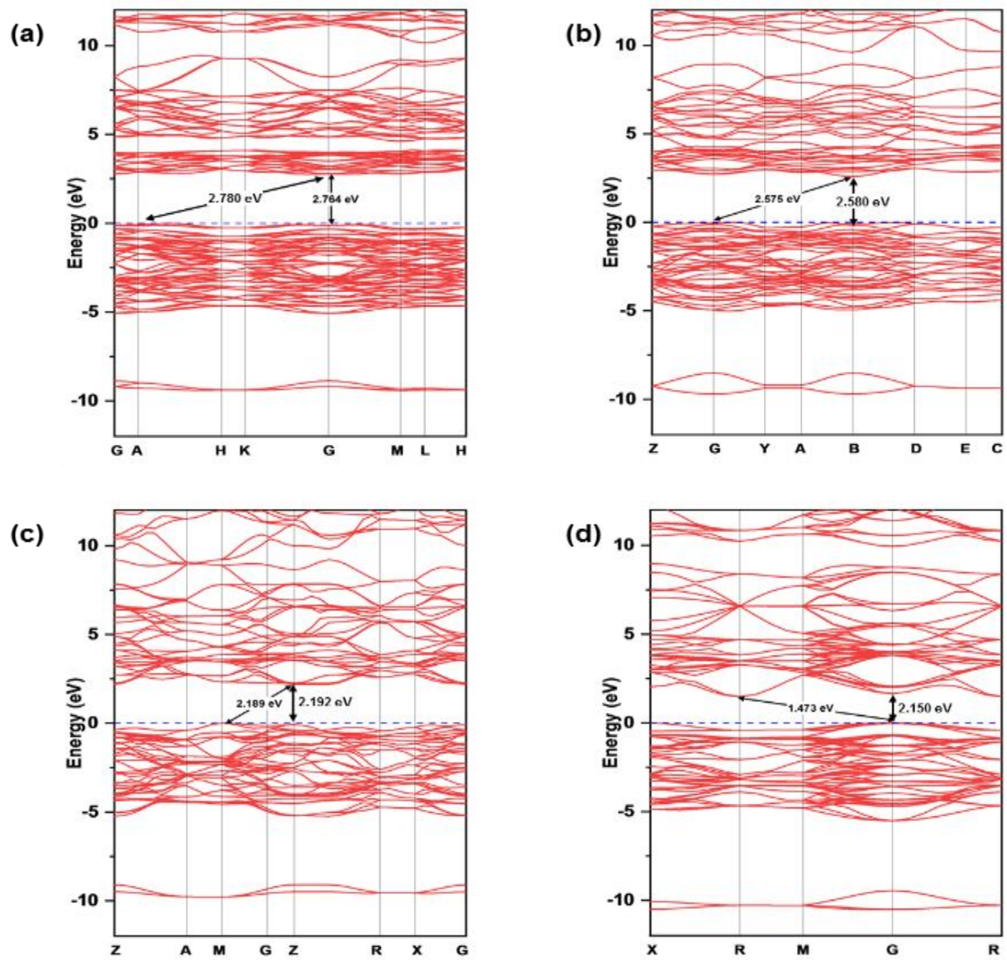

Figure 2. *Band structures of (a) rhombohedral, (b) monoclinic, (c) tetragonal, and (d) cubic BNT phases*

Fig.3 shows the contribution of each cation and anion in the calculated partial densities of state (PDOS) of the four BNT phases. In all the cases, the contributions of *p*-orbitals of Bi, *d*-orbitals of Ti, and *p*-orbitals of O are significant within the low energy range. In addition, the hybridization of *d*-orbitals of Ti and *p*-orbitals of O occurs within the low energy range of approximately -6 eV. In the range above Fermi level, the contribution of Ti *d*-orbitals is appeared to create a boundary of the bandgap; Bi *p*-orbitals also contributed to this range. Still, it is the only evidence in the high-temperature phases.

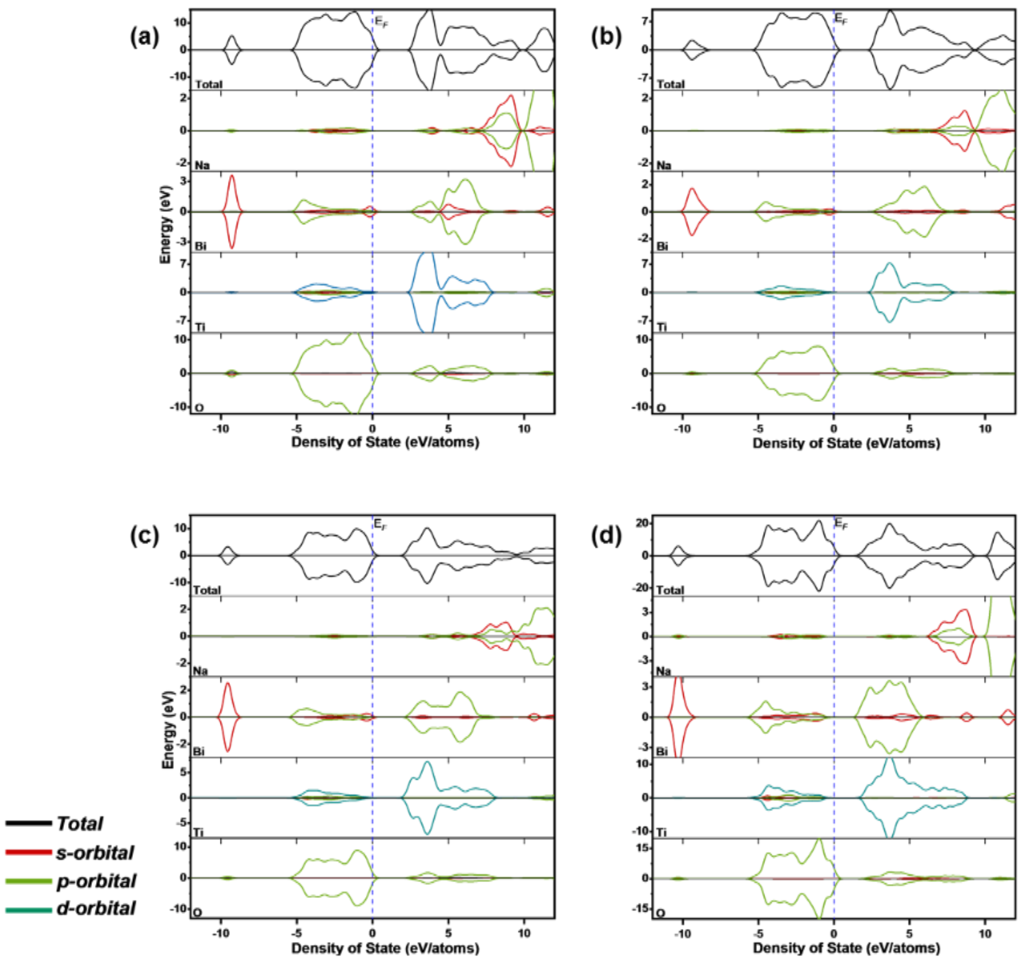

Figure 3. *The densities of state of (a) rhombohedral, (b) monoclinic, (c) tetragonal, and (d) cubic phase of BNT*

4. Conclusion

We have studied the structural and electronic properties of (-$Bi_{0.5}Na_{0.5}TiO_{3-}$) materials in four possible phases using first-principle calculations. The equilibrium structures were determined, and the contributions of *p*-orbitals of Bi and O, and Ti *d*-orbitals are significant within the low energy range. The contribution of *d*-orbitals of Ti is essential creating the bandgap, and *p*-orbitals of Bi are also crucial in the high symmetry phases. The direct bandgap of 2.76 eV and indirect bandgap of 1.5~2.6 eV for the rhombohedral and higher symmetry phases, respectively. Thus, BNT materials have good potential in optical materials. Furthermore, we expected that our works could help to understand the role of phase transition in lead-free ferroelectric sodium bismuth titanate materials.

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INTEGRATED RESOLVENT OPERATORS AND NONDENSELY INTEGRODIFFERENTIAL EQUATIONS INVOLVING THE NONLOCAL CONDITIONS

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Abstract: *The aim of this work is to prove some results of the existence and regularity of solutions for some nondensely integrodifferential equations with nonlocal conditions, where the linear part has an integrated resolvent operator in the sens given by Oka [7]. They extend the results of [4] and [5].*

Keywords: *Integrated resolvent operator, resolvent operator, integral solution, nonlocal, nondensely, integrodifferential equations.*

1. Introduction

Nonlocal conditions in dynamical systems play an important role in many physical problems. They have better effects in applications than the classical initial conditions $u(0) = u_0$. See, for example, in [1,2] to determine the unknown physical parameter in some inverse heat condition problems and in [3] to describe the diffusion phenomenon of a small amount of gas in a transparent tube. As indicated in [8], we sometimes need to deal with non-densely defined operators. For example, when we look at a one-dimensional heat equation with Dirichlet conditions on $[0,\pi]$ and consider $A = \dfrac{\partial^2}{\partial x^2}$ in $C([0,\pi],\mathbb{R})$, in order to measure the solutions in the sup-norm, then the domain.

$$\mathrm{D}(A) = \left\{u \in C^2([0,\pi],\mathbb{R}) : u(0) = u(\pi) = 0\right\}$$

is not dense in $C([0,\pi],\mathbb{R})$ with the sup-norm since

$$\overline{\mathcal{D}(A)} = \left\{u \in C([0,\pi],\mathbb{R}) : u(0) = u(\pi) = 0\right\} \neq C([0,\pi],\mathbb{R}).$$

In this work, we are concerned with the existence and regularity of solutions for the following nondensely nonlocal integrodifferential equation

$$u^{'}(t) = Au(t) + \int_0^t B(t-s)u(s)ds + f(t,u(t)) \qquad \text{for } t \in [0,a] \tag{1.1}$$

$$u(0) = u_0 + g(u)$$

where $A : D(A) \subset X \to X$ is a nondensely defined closed linear operator on a Banach space X, $(B(t))_{t\geq 0}$ is a family of closed linear operators on X having the same domain $D(B) \supset D(A)$ which is independent of t, $f : [0,a] \times X \to X$ and $g : C([0,a]; X) \to X$ are given functions to be specified later, where $C([0,a]; X)$ denotes the space of continuous function form $[0,a]$ to X.

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In [5], Ezzinbi and Liu studied the special case of (1.1) when $B(t) \equiv 0\ \forall t \in [0,a]$. More precisely, they studied the following nonlocal evolution equations

$$\frac{du}{dt} = Au(t) + f(t,u(t)),\ t \in [0,1] \qquad (1.2)$$
$$u(0) = u_0 + g(u),$$

By using the fixed-point methods and the theory of integrated semigroup Ezzinbi and Liu obainted the existence and uniqueness of mild solution to (1.2) when A is not necessarily densely defined but satisfies the Hille-Yosida condition. Then, they verified that mild solutions are ``strict solutions" if additional conditions are assumed.

It is worth emphasizing that in [4] Ezzinbi and Ghnimi proved the existence and regulariy of solutions to (1.1) when A is densely defined and has a resolvent operator in the sens given by Grimmer [6]. However, in the case that the operator A is not densely defined, their results the existence and regulariy are not guaranteed.

2. Integrated resolvent operators

In this section, we summarize basic results which are useful in the sequel. Let Z and W be Banach spaces. We denote by $\mathcal{L}(Z,W)$ the Banach space of bounded linear operators from Z into W endowed with the operator norm, and we abbreviate to $\mathcal{L}(Z)$ when $Z = W$.

Let $A: D(A) \subseteq X \to X$ be a closed linear operator whose domain is not necessarily densely defined in X and $(B(t))_{t\geq 0}$ be a family of linear operators in X with $D(A) \subset D(B(t))$ for $t \geq 0$ and of bounded linear operators from Y into X. Here, Y is the Banach space $D(A)$ equipped with the graph norm $|y|_Y := |Ay| + |y|$ for $y \in Y$. We start by putting together the fundamental properties on integrated resolvent operators. We refer to Oka [7] for more details. Let us consider the following integrodifferential equation:

$$\begin{cases} x'(t) = Ax(t) + \int_0^t B(t-s)x(s)\mathrm{d}s \quad \text{for } t \geq 0 \\ x(0) = x_0 \in X \end{cases} \qquad (2.1)$$

Definition 2.1. ([7]) An integrated resolvent operator for Eq.(2.1) is a bounded operator-valued function $R(t) \in \mathcal{L}(X)$ for $t \geq 0$, having the following properties.

r_1: For all $x \in X, R(.)x \in C([0,+\infty); X)$.

r_2: For all $x \in X, \int_0^{\cdot} R(s)xds \in C([0,+\infty); Y)$.

r_3: $R(t)x - tx = A\int_0^t R(s)x\,ds + \int_0^t B(t-s)\int_0^s R(r)xdrds$ for all $x \in X$ and $t \geq 0$.

r_4: $R(t)x - tx = \int_0^t R(s)Axds + \int_0^t\int_0^s R(s-r)B(r)xdrds$ for all $x \in D(A)$ and $t \geq 0$.

Remark 2.2. Definition 2.1 generalizes that of integrated semigroup of A when $B = 0$

Definition 2.3. An integrated resolvent operator $(R(t))_{t\geq 0}$ in $\mathcal{L}(X)$ is called locally Lipschitz continuous, if for all $a > 0$, there exists a constant $C_a = C(a) > 0$ such that:

$$|R(t) - R(s)| \leq C_a |t - s| \text{ for } t, s \in [0, a].$$

Theorem 2.4. ([7]) Suppose that $(R(t))_{t\geq 0}$ is a locally Lipschitz continuous integrated resolvent operator. Then for all $x \in \overline{D(A)}, t \to R(t)x$ is a C^1-function on $[0, +\infty)$.

We now introduce the following assumptions:

(H0): The operator A satisfies the Hille-Yosida condition.

(H1): $(B(t))_{t\geq 0}$ is a family linear operator in X with $D(A) \subset D(B(t))$ for all $t \geq 0$ and, of bounded linear operators from Y to X such that the functions $B(.)x$ are of strong bounded variation on each finite interval $[0, a], a > 0$, for $x \in D(A)$.

The following result provides sufficient conditions ensuring the existence of locally Lipschitz continuous integrated resolvent operator for Eq.(2.1).

Theorem 2.5. ([7]) *Assume that* **(H0)** *and* **(H1)** *hold. Then, there exists a unique locally Lipschitz continuous integrated resolvent operator of Eq.*(2.1).

We study the following initial value problem:

$$\begin{cases} x'(t) = Ax(t) + \int_0^t B(t-s)x(s)ds + q(t) \quad \text{for } t \geq 0 \\ x(0) = x_0 \end{cases} \tag{2.2}$$

where $x_0 \in X$ and $q \in C([0, +\infty); X)$. We shall introduce the notions of integral and strict solutions to Eq.(2.2) and give some results concerning the existence and regularity of solutions of Eq.(2.2) used in the later sections.

Definition 2.6. ([7]) Let $q \in L^1_{\text{loc}}(0, +\infty; X)$ and $x_0 \in X$. A function $x : [0, +\infty) \to X$ is called an integral solution of Eq.(2.2) if the following conditions hold:

i) $x \in C([0, +\infty); X)$.

ii) $\int_0^{\cdot} x(s)ds \in C([0, +\infty); Y)$.

iii) $x(t) = x_0 + A\int_0^t x(s)ds + \int_0^t B(t-s)\int_0^s x(r)drds + \int_0^t q(s)ds$ for $t \geq 0$.

Remark 2.7. If x is an integral solution of Eq. (1.1) then, it follows from Definition 2.6. that $x(t) \in \overline{D(A)}$ for all $t \geq 0$. Indeed, $x(t) = \lim_{h\to 0} \frac{1}{h}\int_t^{t+h} x(s)ds$ and $\int_t^{t+h} x(s)ds \in D(A)$. In particular, $x(0) \in \overline{D(A)}$ is a necessary condition for existence of an integral solution of Eq.(2.2).

Definition 2.8. A function $x : [0, +\infty) \to X$ is called a strict solution of Eq.(2.2) if the following conditions hold:

i) $x \in C^1([0,+\infty);X) \cap C([0,+\infty);Y)$.

ii) x satisfies Eq.(2.2) on $[0,+\infty)$.

Theorem 2.9. ([7]) *Assume that* $\overline{D(A)} = X$ *and* $\rho(A) \neq \varnothing$. Let $q \in L^1_{\text{loc}}(0,+\infty;X)$. *The following statements are equivalent*

i) *Eq.*(2.1) *admits a locally Lipschitz continuous integrated resolvent operator* $(R(t))_{t\geq 0}$.

ii) *Eq.* (2.1) *admits a resolvent operator* $(R(t))_{t\geq 0}$.

iii) *For all* $x_0 \in X$ *there exists a unique integral solution* x *to* Eq.(2.2).

iv) *For all* $x_0 \in X$ *, there exists a unique weak solution* x *to Eq.*(2.2). *In this case,*

$$R(t)x_0 = \int_0^t R(s)x_0 ds \text{ for } t \geq 0 \text{ and } x_0 \in X$$

$$x(t) = \frac{d}{dt}\left(R(t)x_0 + \int_0^t R(t-s)q(s)ds\right) \text{ for } t \geq 0$$

$$= R(t)x_0 + \int_0^t R(t-s)q(s)ds \text{ for } t \geq 0 \text{ and } x_0 \in X$$

The following is a key Theorem to prove our main results.

Theorem 2.10 ([7]). *Let a family* $(U(t))_{t\geq 0}$ *in* $\mathcal{L}(X)$ *be locally Lipschitz continuous with* $U(0)=0$. *Then, the following holds:*

i) If $q \in L^1(0,a;X)$ *, then* $\int_0^t U(.-s)q(s)ds \in C^1([0,a];X)$.

Putting $Q(t) := \frac{d}{dt}\int_0^t U(t-s)q(s)ds$ *for* $t \in [0,a]$, *we have*

$$|Q(t)| \leq C_a \int_0^t |q(s)|\, ds$$

where C_a *is the Lipschitz constant of* $U(t)$ *on* $[0,a]$. *Moreover, if* $|q(t)| \leq K$ *for* $t \in [0,a]$, *then*

$$|Q(t+s) - Q(t)| \leq KC_a s + C_a \int_0^t |q(s+r) - q(r)|\, dr \text{ for } s,t,t+s \in [0,a].$$

ii) *If a function* $q:[0,a] \to X$ *is of strong bounded variation, the function* $Q(.)$ *defined in* i) *is Lipschitz continuous on* $[0,a]$.

Remark 2.11. The results reported in Theorem 2.10 hold for any locally Lipschitz continuous family of bounded linear operators $(U(t))_{t\geq 0}$ with $U(0)=0$. In particular, these results are true for the integrated resolvent operators.

The following theorem gives sufficient conditions for the existence of integral and strict solutions of Eq.(2.2).

Theorem 2.12. ([7]) *Assume that Eq.*(2.1) *has an integrated resolvent operator* $(R(t))_{t\geq 0}$ *that is locally Lipschitz continuous and* $\rho(A) \neq \varnothing$. *Then, the following holds.*

i) *If* $x_0 \in \overline{D(A)}$ *and* $q \in L^1(0,a;X)$, *then there exists a unique integral solution* $x(.)$ *of Eq.* (2.2) *which is given by the variation of constants formula*

$$x(t) = R'(t)x_0 + \frac{d}{dt}\int_0^t R(t-s)q(s)ds \quad \text{for } t \in [0,a].$$

Moreover, we have $|x(t)| \le C_a\left(|x_0| + \int_0^t |q(s)|\,ds\right)$ for $t \in [0,a]$.

ii) *If* $x_0 \in D(A), q \in W^{1,1}(0,a;X)$ *and* $Ax_0 + q(0) \in \overline{D(A)}$, *then there exists a unique strict solution* $x(.)$ *of Eq.*(2.2). *Moreover, we have*

$$\left| x'(t) \right| \le C_a\left(\left|Ax_0 + q(0)\right| + \int_0^t \left|B(s)x_0 + q'(s)\right| ds \right) \quad \text{for } t \in [0,a].$$

3. Existence and Regularity of Solutions

Definition 3.1. A continuous function $u:[0,a] \to X$ is said to be a strict solution of Eq.(1.1) if

i) $u \in C^1([0,a];X) \cap C([0,a];Y)$,

ii) u satisfies Eq.(1.1).

Definition 3.2. A continuous function $u:[0,a] \to X$ is said to be a mild solution of Eq.(1.1) if

$$u(t) = R'(t)\big(u_0 + g(u)\big) + \frac{d}{dt}\int_0^t R(t-s)f(s,u(s))ds \text{ for } t \in [0,a].$$

To prove the existence of mild solutions, we make the following assumptions:

(**H2**): Function $f:[0,a]\times X \to X$ is continuous and Lipschitzian with respect to the second argument. Let $L_f > 0$ be such that

$$|f(t,u) - f(t,v)| \le L_f\,|u - v| \text{ for } t \in [0,a] \text{ and } u,v \in X.$$

(**H3**): Function $g: C([0,a];X) \to X$ is Lipschitz continuous. Let $L_g > 0$ be such that

$$|g(u) - g(v)| \le L_g\,|u - v| \text{ for } u,v \in C([0,a];X).$$

Theorem 3.3. *Assume that Eq.* (2.1) *has an integrated resolvent operator* $(R(t))_{t\ge 0}$ *that is locally Lipschitz continuous and* $\rho(A) \ne \emptyset$. *Let* f, g *be two functions satisfying* (**H2**) *and* (**H3**) *respectively, and* $u_0 + g(u) \in \overline{D(A)}$. *Then the nonlocal problem* (1.1) *has a unique mild solution* u *on* $[0,a]$ *provided that*

$$C_a\left(L_g + L_f a\right) < 1. \tag{3.1}$$

Proof. Consider the operator $\Phi: C([0,a];X) \to C([0,a];X)$ defined by

$$(\Phi u)(t) = R'(t)\left[u_0 + g(u)\right] + \frac{d}{dt}\int_0^t R(t-s)f(s,u(s))ds \quad \text{for} \quad t \in [0,a].$$

Let $u, v \in C([0,a];X)$. Then for $t \in [0,a]$, we have

$$|(\Phi u)(t)-(\Phi v)(t)| \leq |R'(t)[g(u)-g(v)]|+\left|\frac{d}{dt}\int_0^t |R(t-s)[f(s,u(s))-f(s,v(s))]|\,ds\right|$$
$$\leq C_a\left(L_g|u-v|+L_f\int_0^t |u(s)-v(s)|\,ds\right)$$
$$\leq C_a\left(L_g+L_f a\right)|u-v|,$$

which implies that $|(\Phi u)-(\Phi v)| \leq M_a\left(L_g+L_f a\right)|u-v|$.

Thus, from (3.1), Φ is a strict contraction. Then by the Banach's fixed point theorem Φ has a unique fixed point in $C([0,a];X)$, which means there exists a unique mild solution for Equation (1.1) on $[0,a]$. For the regularity of the mild solution, we assume the following assumption:

(**H4**): $f \in C^1([0,a]\times X;X)$ and the partial derivatives $D_1 f(.,.)$ and $D_2 f(.,.)$ are locally Lipschitzian with respect to the second argument.

Theorem 3.4. *Assume that Equation* (2.1) *has an integrated resolvent operator* $(R(t))_{t\geq 0}$ *that is locally Lipschitz continuous and* $\rho(A)\neq\varnothing$. *Let* (**H2**) - (**H4**) *hold and*

$$u(0)=u_0+g(u)\in D(A) \tag{3.2}$$
$$Au(0)+f(0,u(0))\in \overline{D(A)}.$$

Then, the integral solution of Equation (1.1) *given by* Theorem 3.3 *is a strict solution on* $[0,+\infty)$.

Proof. Let u be the mild solution of Equation (1.1) given by Theorem 3.3. Then

$$u(t)=R'(t)u(0)+\frac{d}{dt}\int_0^t R(t-s)f(s,u(s))ds \text{ for } t\in[0,a]. \tag{3.3}$$

Differentiating (r_4) with $x=u(0)\in D(A)$, we get

$$R'(t)u(0)=.(0)+R(t)Au(0)+\int_0^t R(t-s)B(s)u(0)ds \quad \text{for } t\in[0,a].$$

which implies, together with (3.3), that

$$u(t)=u(0)+R(t)Au(0)+\int_0^t R(t-s)B(s)u(0)ds \tag{3.4}$$
$$+\frac{d}{dt}\int_0^t R(t-s)f\left(s,u(s)\right)ds \quad \text{for } t\in[0,a].$$

Consider now the following equation

$$\frac{dv(t)}{dt}=Av(t)+\int_0^t B(t-s)v(s)ds+D_1 f(t,u(t))+D_2 f(t,u(t))v(t) \tag{3.5}$$
$$+B(t)u(0) \text{ for } t\in[0,a]$$
$$v(0)=A\,u(0)+f\left(0,\ u(0)\right).$$

where D_1 and D_2 are the partial derivatives to the first and second variables, respectively.

Then by the contraction mapping principle we can prove that equation (3.5) has an integral solution v on $[0,T]$ which is given by

$$v(t) = R'(t)\left[A\,u(0) + f\left(0,\ u(0)\right)\right] \tag{3.6}$$

$$+\frac{d}{dt}\int_0^t R(t-s)\left[D_1 f(s,u(s)) + D_2 f(s,u(s))v(s) + B(s)u(0)\right]ds.$$

Let w the function be defined by

$$w(t) = u(0) + \int_0^t v(s)ds \text{ for } t \in [0,a].$$

Now, we shall prove that $w = u.$ In view of (3.2) it follows from (3.6) that the solution v of Eq. (3.5) satisfies

$$v(t) = R'(t)\left[A\,u(0) + f\left(0,\ u(0)\right)\right]$$

$$+\frac{d}{dt}\int_0^t R(t-s)\left[D_1 f(s,u(s)) + D_2 f(s,u(s))v(s) + B(s)u(0)\right]ds.$$

Integrating this over $[0,t]$, we obtain

$$\int_0^t v(s)ds = R(t)[Au(0) + f(0,u(0))]$$

$$+\int_0^t R(t-s)\left[D_1 f(s,u(s)) + D_2 f(s,u(s))v(s) + B(s)u(0)\right]ds$$

and so

$$R(t)Au(0) = -R(t)f(0,u(0)) + \int_0^t v(s)ds \tag{3.7}$$

$$-\int_0^t R(t-s)\left[D_1 f(s,u(s)) + D_2 f(s,u(s))v(s) + B(s)u(0)\right]ds.$$

On the other hand, since the function $t \to w(t)$ is continuously differentiable, it follows from Theorem 2.10 that

$$t \to \int_0^t R(t-s) f\left(s, w(s)\right) ds$$

is also continuously differentiable and

$$\frac{d}{dt}\int_0^t R(t-s) f\left(s,w(s)\right) ds = \frac{d}{dt}\int_0^t R(s) f\left(t-s, w(t-s)\right) ds$$

$$= R(t)\ f\left(0,u(0)\right) + \int_0^t R(t-s)\left[D_1 f\left(s,w(s)\right) + D_2 f\left(s,w(s)\right)v(s)\right]ds.$$

This implies that

$$R(t) f(0,u(0)) = \frac{d}{dt}\int_0^t R(t-s) f\left(s,w(s)\right) ds \tag{3.8}$$

$$-\int_0^t R(t-s)\left[D_1 f\left(s,w(s)\right) + D_2 F\left(s,w(s)\right)v(s)\right]ds.$$

Combining (3.4) with (3.7) and (3.8), we find

$$
\begin{aligned}
u(t) &= u(0) - R(t)f(0,u(0)) + \int_0^t v(s)ds \\
&\quad - \int_0^t R(t-s)\left[D_1 f(s,u(s)) + D_2 f(s,u(s))v(s) + B(s)u(0)\right]ds \\
&\quad + \int_0^t R(t-s)B(s)u(0)ds + \frac{d}{dt}\int_0^t R(t-s)f\left(s,u(s)\right)ds \\
&= w(t) - \frac{d}{dt}\int_0^t R(t-s)f\left(s,w(s)\right)ds \\
&\quad + \int_0^t R(t-s)\left[D_1 f\left(s,w(s)\right) + D_2 F\left(s,w(s)\right)v(s)\right]ds \\
&\quad - \int_0^t R(t-s)\left[D_1 f(s,u(s)) + D_2 f(s,u(s))v(s) + B(s)u(0)\right]ds \\
&\quad + \int_0^t R(t-s)B(s)u(0)ds + \frac{d}{dt}\int_0^t R(t-s)f\left(s,u(s)\right)ds \\
&= w(t) + \frac{d}{dt}\int_0^t R(t-s)\left[f(s,u(s)) - f(s,w(s))\right]ds \\
&\quad - \int_0^t R(t-s)\left[D_1 f(s,u(s)) - D_1 f(s,w(s))\right]ds \\
&\quad - \int_0^t R(t-s)\left[D_2 f(s,u(s)) - D_2 f(s,w(s))\right]v(s)ds.
\end{aligned}
$$

Consequently,

$$
\begin{aligned}
u(t) - w(t) &= \frac{d}{dt}\int_0^t R(t-s)\left[f(s,u(s)) - f(s,w(s))\right]ds \\
&\quad - \int_0^t R(t-s)\left[D_1 f(s,u(s)) - D_1 f(s,w(s))\right]ds \\
&\quad - \int_0^t R(t-s)\left[D_2 f(s,u(s)) - D_2 f(s,w(s))\right]v(s)ds
\end{aligned}
$$

and so,

$$
\begin{aligned}
|u(t) - w(t)| &\le \left|\frac{d}{dt}\int_0^t R(t-s)\left[f(s,u(s)) - f(s,w(s))\right]ds\right| \\
&\quad + \left|\int_0^t R(t-s)\left[D_1 f(s,u(s)) - D_1 f(s,w(s))\right]ds\right| \\
&\quad + \left|\int_0^t R(t-s)\left[D_2 f(s,u(s)) - D_2 f(s,w(s))\right]v(s)ds\right|.
\end{aligned}
$$

Let $\mathcal{K}=\{u(t),w(t):t\in[0,a]\}$. Then $\mathcal{K}$ is a compact set. Since $D_1 f$ and $D_2 f$ are locally Lipschitz with respect to the second argument, then $D_1 f$ and $D_2 f$ are globally Lipschitz on $\mathcal{K}$. Thus there exists $\gamma(a)>0$ such that

$$|u(t)-w(t)|\leq\gamma(a)\int_0^t|u(s)-v(s)|\,ds \text{ for } t\in[0,a]$$

where $\gamma(a)=C_a L_f+b_0\operatorname{Lip}\left(D_1 f\right)+b_0^2\operatorname{Lip}\left(D_2 f\right)$

with $b_0:=\max\left\{\sup\limits_{0\leq s\leq a}|R(s)|,\sup\limits_{0\leq s\leq a}\left|v(s)\right|\right\}.$

By Gronwall's lemma, we deduce that $u(t)=w(t)$ for $t\in[0,a]$. Then u is continuously differentiable in $[0,a]$. So the function $t\to f(t,u(t))$ is continuously differentiable on v, which means, by Theorem 2.12 that u is a strict solution of Eq.(1.1) on $[0,a]$.

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ISOLATION, IDENTIFICATION AND OPTIMIZATION OF MASS PRODUCTION OF *Bacillus velezensis*

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Abstract: *Bacillus velezensis strain BL-1 was successfully isolated from the soil samples collected from Quang Xuong district, Thanh Hoa province, Vietnam. The BL-1 strain was identified as Bacillus velezensis based on morphological and phylogenetic analysis of its 16S rRNA gene. Our study proposed a protocol for mass production of B. velezensis BL-1 using a soybean flour molasses-based medium which provided a high efficiency. The optimized fermentation conditions were an incubation for 48 hours at 35°C with a shaker at speed of 120 rpm, pH 7.0, and inoculum volume of 2% (v/v).*

Keywords: *Bacillus velezensis, isolation, mass production.*

1. Introduction

Developing organic agricultural production is an inevitable direction in Vietnam and other countries in the world to ensure sustainable, harmonious, nature-friendly, and safe agriculture. Since biopesticides and microbiological organic fertilizers play an important role in the success of organic agricultural production, investigation, and application of microorganisms have become more widespread [8].

The bacterial genus *Bacillus* has a wide range of physiological properties and their ability to produce a various range of enzymes, antibiotics and metabolites have made them potential agents for agricultural applications. *Bacillus velezensis*, a typical of *Bacillus* species, is an endospore-forming, free-living soil bacterium with potentials as a plant growth-promoting agent, a biopesticide against a broad spectrum of microbial pathogens of plants, and an important source of microorganisms for producing organic fertilizers [10].

Many strains of *B. velezensis* have widely been reported to benefit plant growth by nutrient uptake and secreting secondary metabolites such as indole-3-acetic acid to promote the system development of plant roots [4] [6]. Numerous strains of this species have been reported to suppress the growth of microbial pathogens, including bacteria, fungi, and nematodes [10]. In Vietnam, researches on the isolation and selection of *Bacillus* sp. for agricultural applications have been carried out widely in recent years. Three strains of bacteria belonging to the species *Bacillus amyloliquefaciens, Bacillus velezensis,* and *Bacillilus subtilis* were isolated and characterized with capacities of degrading insoluble nutrients in the soil, decomposing cellulose quickly, and growing well in the composting process in Binh Thuan province [12]. *B. velezensis* showed high inhibitory efficiency against fungal pathogen *Phytophthora* sp. [13].

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In the present study, we attempt to isolate and identify a strain of *B. velezensis* that performs promising properties for agricultural application in Thanh Hoa province, Vietnam. The optimal conditions for mass production of strain *B. velezensis* are also examined to facilitate the production of microbiological fertilizers.

2. Methods

2.1. Soil sample collection and isolation of Bacillus sp.

Fifteen grams of soil sample from the root zone at depth of 5-10 cm were collected using a sterile inoculating spoon in Quang Xuong district, Thanh Hoa province, Vietnam. Each sample was packaged in a sterile bottle, labeled appropriately. The samples were stored at 4-8^{o}C until ready for processing. Ten grams of each soil sample were suspended in 90 mL of sterilized distilled water (SDW) in universal bottles. The soil suspension was heat-shocked at 60^{o}C for one hour in a water bath to kill non-spore-forming organisms [14]. The samples were shaken for 10 minutes (min) at 150 rpm and left to stand for 10 minutes afterward. Then 3 serial 10-fold dilutions were made. Finally, 1 mL of the serially-diluted suspensions was spread on Nutrient Agar (peptone 5 g/L, yeast extracts 3 g/L and agar 20 g/L, pH 7,0-7,2). Petri plates were sealed and incubated at 37^{o}C for 24h and examined for colony morphology. The colonies that exhibited typical characteristics of *Bacillus* species (round or irregular thick and opaque; cream-colored colonies) were sub-cultured onto nutrient agar plates for further screening to obtain pure cultures [1].

2.2. Identification of Bacillus sp. by 16S rRNA gene sequence analysis

Genes of 16S rRNA were sequenced and blasted for identification of the bacterial species. The bacterial cells were harvested for chromosomal DNA isolation and purification. The 16S rRNA gene was amplified via the polymerase chain reaction (PCR) following the protocol provided by Bio-techem CO., LTD (www.biotechem.com.vn). After the amplification was completed, the fragment was detected by agarose gel electrophoresis, recovered, and sequenced. The 16S rRNA sequences were confirmed and compared using a BLAST nucleotide search provided by the National Center for Biotechnology Information (NCBI) GenBank (U.S. National Library of Medicine, Bethesda, Maryland, USA). The phylogenetic tree was constructed using the maximum likelihood method by MEGA 5.1 software [11].

2.3. Identification of the optimal conditions for mass production of Bacillus sp.

For optimal carbon source: To standardize the medium for the optimum growth of *Bacillus* sp., a preliminary shake flask culturing was conducted to select the most suitable carbon source. The conical flask was added with 10 g of carbon source (either Glucose, Sucrose, Lactose or Glycerol), 10 g yeast extract, 1.5 g NaCl, K2HPO4 1.5 g, MgSO4 1.5 g per liter, pH 7.0 and incubated on a shaker at speed of 120 rpm, 35^{o}C for 48 hours. The effective bacterial growth was tested via serial dilutions, spreading diluted suspensions of

Bacillus sp. culture on plates with media containing different sources of carbon, and incubated for 24 hours.

For optimal nitrogen source: Three nitrogen sources including Soybean flour, Yeast extract, and Peptone were examined for the selection of the best one. The batch process was started with an initial volume of 2 L of either sterile molasses soybean flour (molasses 10%, soybean four 2%), molasses yeast extract (molasses 10%, yeast extract 1%), or molasses peptone (molasses 10%, peptone 1%) media in pH 7. They were inoculated with a 25 mL shake flask pre-inoculated *B. velezensis* BL-1 culture at 35^{o}C on a shaker at 120 rpm to agitate the culture. The optimum aeration was standardized using an aquarium air pump, 8-Liter/ min (VIPSUN, China). The effective bacterial growth in each tested medium was tested by serial dilution and spreading *B. velezensis* BL-1 culture on plates for 24 hours.

For optimal duration of batch fermentation: Batch cultures were carried out in a fermentor with 5 mL capacity. Two liters of Soybean flour molasses medium was sterilized, then 40 mL of this pre-inoculated bacterial culture broth (10^6 CFU/mL) was inoculated into 2 L of King's B broth or soybean flour liquid, followed by incubation at 35^{o}C on a shaker at 120 rpm to agitate the culture, under aerobic condition (pumping fresh air) for 2 days. The viable cell density of bacteria was counted at 12, 24, 48 hours after incubation by serial dilution and spread planting of *B. velezensis* BL-1 culture for 24 hours.

2.4. General procedure for production of Bacillus sp.

Bacillus sp. were streaked onto King's B medium (peptone 15 g/L, K_2HPO_4 1.5 g/L, $MgSO_4$ 15 g/L, glycerol 10 mL/L, agar 20g/L, pH 7,0 – 7,2) and incubated at 35^{o}C for 2 days.

Bacterial liquid cultures were carried out in a 250 mL flask with a working volume of 100 mL of King'B broth. Transfer a loopful of *Bacillus* sp. culture to 100 mL of sterilized King's B broth in a 250 mL conical flask and incubate on a shaker at 120 rpm, 35^{o}C for 48 hours. The number of cells was counted as colony-forming units (CFUs) by making serial dilutions, spreading suspensions on King's B medium plate, and incubating at 35^{o}C for 1 day. Cell density was adjusted to the optimum for subsequent use.

Batch cultures were carried out in a fermentor of 5 L capacity, prepare 2 L of King's B broth or soybean flour molasses broth, and sterilize. Then, 40 mL of this pre-inoculated bacterial culture (10^6 CFUs/mL) was inoculated into 2 L of King's B broth or soybean flour liquid, followed by incubation at 35^{o}C, under aerobic condition (pumping fresh air) for 2 days. The viable cell density (CFUs/mL) of the culture was determined and adjusted for use.

Sterilize substrate (talc powder) at 121^{o}C, 1 atm for 30 min in two successive days. Transfer 1 kg of the sterilized substrate into a polythene bag or sterile container under aseptic conditions and add bacterial broth culture to reach the cell density of 1 - 2 x 10^8 CFUs/g. Mix thoroughly and shade dry to bring its moisture content to less than 20%. The formulation was packed into a polythene bag and could be stored for 3 - 4 months under room temperature (RT).

3. Results and discussion

3.1. Morphological characterization

Colonies of bacterial isolate BL-1 were flat on King'B solid medium, with a serrated edge, dirty white, rough surface, opaque, viscous, and 3–5 mm in diameter at 30^{o}C for 1 day after inoculation (Fig. 1A). Gram staining showed blue-purple stained cells, indicating that the isolate BL-1 was Gram-positive (Fig. 1B). The results indicated that isolate BL-1 had close morphology with the genus *Bacillus*, which made the preliminary reference for the next step of identification by sequence homology among published reference sequences with the BLAST tool.

(A) **(B)**

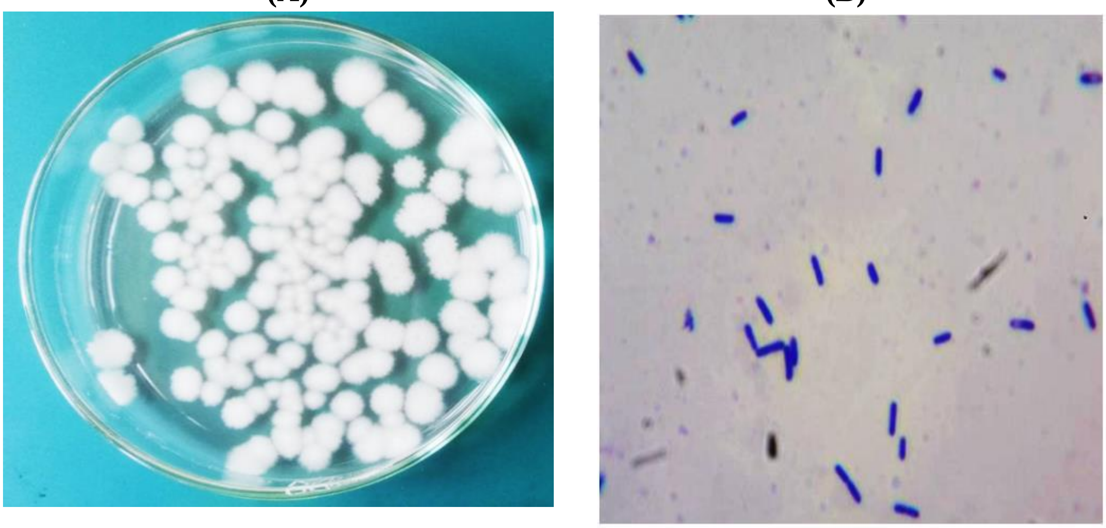


Figure 1. *Colony growth of BL-1 on King'B medium after 1 day inoculation (A) and Gram-positive cells under the microscope (**B**)*

3.2. Genetic identification of isolate BL-1 based on phylogenetic analysis of 16S rRNA

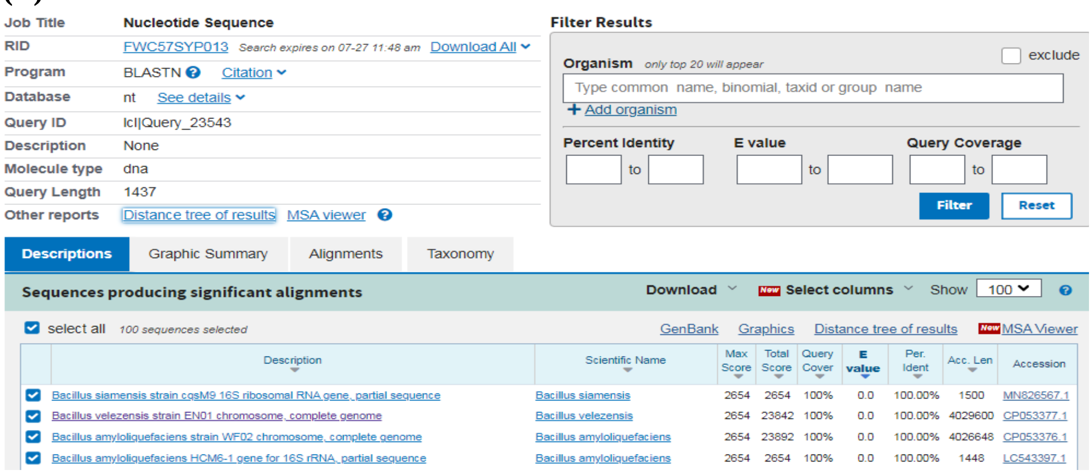
A successful PCR yields a single band of the expected size of 16S rRNA gene of isolate BL-1 that was purified and then sequenced (Fig. 2A). The size of 16S rRNA gene of isolate BL-1 was determined as 1437 bp. The gene sequence of BL-1 was compared with sequences deposited in the 16S rRNA database, which resulted in identification at the species level with an identity at 100% with a known *Bacillus velezensis* strain EN01 (CP053377.1) (Fig. 2A, B).

The genetic distance scale is the reference of each branch length in the phylogenetic tree (Fig. 3). The location of the nodes represents the genetic relationship. The shorter the branch length between two end nodes means the relationship is closer. Based on the consistency between the results of the 16S rRNA sequence analysis and the physiological characterization, isolate BL-1 was indicated as a strain of *Bacillus velezensis* BL-1.

(A) 16S rRNA gene sequence of isolate BL-1 (1437bp)

CCCCAATCATCTGTCCCACCTTCGGCGGCTGGCTCCATAAAGGTTACCTCAC
CGACTTCGGGTGTTACAAACTCTCGTGGTGTGACGGGCGGTGTGTACAAGG
CCCGGGAACGTATTCACCGCGGCATGCTGATCCGCGATTACTAGCGATTCC
AGCTTCACGCAGTCGAGTTGCAGACTGCGATCCGAACTGAGAACAGATTTG
TGGGATTGGCTTAACCTCGCGGTTTCGCTGCCCTTTGTTCTGTCCATTGTAG
CACGTGTGTAGCCCAGGTCATAAGGGGCATGATGATTTGACGTCATCCCCA
CCTTCCTCCGGTTTGTCACCGGCAGTCACCTTAGAGTGCCCAACTGAATGCT
GGCAACTAAGATCAAGGGTTGCGCTCGTTGCGGGACTTAACCCAACATCTC
ACGACACGAGCTGACGACAACCATGCACCACCTGTCACTCTGCCCCCGAAG
GGGACGTCCTATCTCTAGGATTGTCAGAGGATGTCAAGACCTGGTAAGGTT
CTTCGCGTTGCTTCGAATTAAACCACATGCTCCACCGCTTGTGCGGGCCCCC
GTCAATTCCTTTGAGTTTCAGTCTTGCGACCGTACTCCCCAGGCGGAGTGCT
TAATGCGTTAGCTGCAGCACTAAGGGGCGGAAACCCCCTAACACTTAGCAC
TCATCGTTTACGGCGTGGACTACCAGGGTATCTAATCCTGTTCGCTCCCCAC
GCTTTCGCTCCTCAGCGTCAGTTACAGACCAGAGAGTCGCCTTCGCCACTGG
TGTTCCTCCACATCTCTACGCATTTCACCGCTACACGTGGAATTCCACTCTC
CTCTTCTGCACTCAAGTTCCCCAGTTTCCAATGACCCTCCCCGGTTGAGCCG
GGGGCTTTCACATCAGACTTAAGAAACCGCCTGCGAGCCCTTTACGCCCAA
TAATTCCGGACAACGCTTGCCACCTACGTATTACCGCGGCTGCTGGCACGT
AGTTAGCCGTGGCTTTCTGGTTAGGTACCGTCAAGGTGCCGCCCTATTTGAA
CGGCACTTGTTCTTCCCTAACAACAGAGCTTTACGATCCGAAAACCTTCATC
ACTCACGCGGCGTTGCTCCGTCAGACTTTCGTCCATTGCGGAAGATTCCCTA
CTGCTGCCTCCCGTAGGAGTCTGGGCCGTGTCTCAGTCCCAGTGTGGCCGAT
CACCCTCTCAGGTCGGCTACGCATCGTCGCCTTGGTGAGCCGTTACCTCACC
AACTAGCTAATGCGCCGCGGGTCCATCTGTAAGTGGTAGCCGAAGCCACCT
TTTATGTCTGAACCATGCGGTTCAGACAACCATCCGGTATTAGCCCCGGTTT
CCCGGAGTTATCCCAGTCTTACAGGCAGGTTACCCACGTGTTACTCACCCGT
CCGCCGCTAACATCAGGGAGCAAGCTCCCATCTGTCCGCTCGAC

(B) Blast results



Description	Scientific Name	Max Score	Total Score	Query Cover	E value	Per. Ident	Acc. Len	Accession
Bacillus siamensis strain cqsM9 16S ribosomal RNA gene, partial sequence	Bacillus siamensis	2654	2654	100%	0.0	100.00%	1500	MN826567.1
Bacillus velezensis strain EN01 chromosome, complete genome	Bacillus velezensis	2654	23842	100%	0.0	100.00%	4029600	CP053377.1
Bacillus amyloliquefaciens strain WF02 chromosome, complete genome	Bacillus amyloliquefaciens	2654	23892	100%	0.0	100.00%	4026648	CP053376.1
Bacillus amyloliquefaciens HCM6-1 gene for 16S rRNA, partial sequence	Bacillus amyloliquefaciens	2654	2654	100%	0.0	100.00%	1448	LC543397.1

Figure 2*. Identification of isolate B. velezensis strain BL-1. (****A****) The 16S rRNA gene sequence of isolate BL-1; (****B****) The 16S rRNA sequences were confirmed and compared through a BLAST nucleotide search provided by NCBI GenBank.*

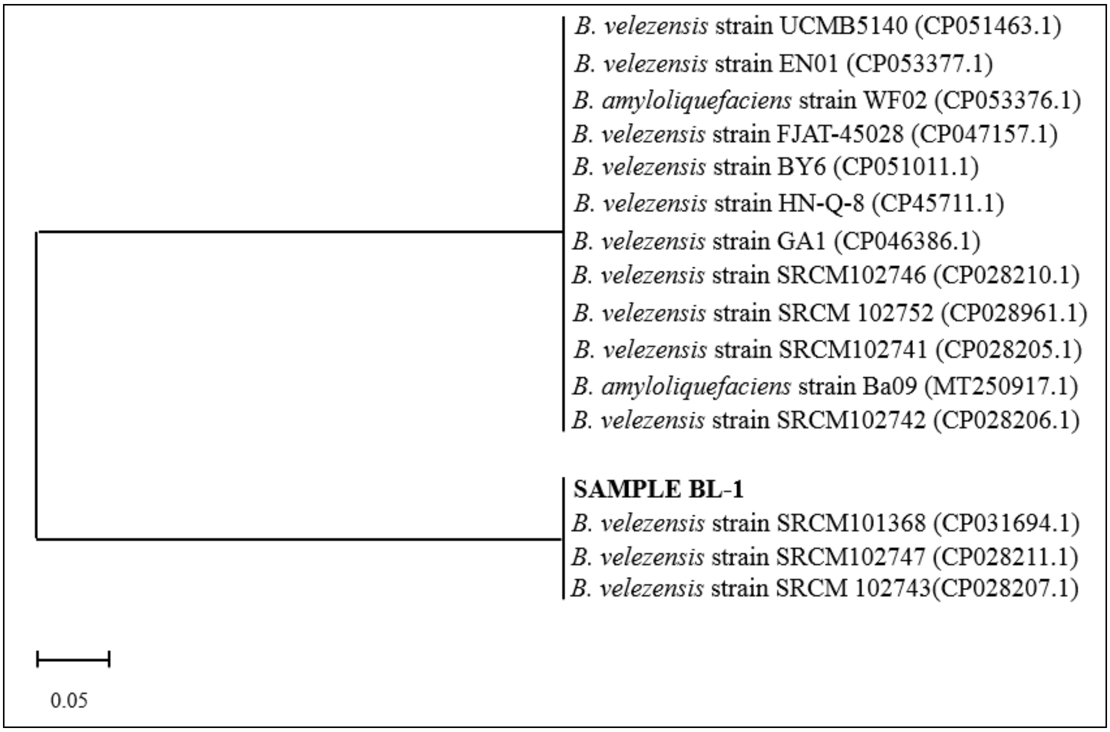


***Figure 3**. Phylogenetic tree based on the partial 16S rRNA gene of BL-1 and other homologous Bacillus strains. The bar represents 0.05 substitutions per site*

3.3. Identification of the optimal conditions for BL-1 strain

3.3.1. Identification of the optimal carbon source in shake flask culturing for BL-1 strain

Four carbon sources (either Glucose, Sucrose, Lactose or Glycerol) were examined for the optimum growth of strain *B. velezensis* BL-1. Among the fourth tested carbon sources for culturing *B. velezensis* BL-1, number of the viable cells cultured on sucrose medium was 4.24 x 10^8 CFU/mL, on average, which was significantly higher than that on other media. Therefore, sucrose was selected as an ideal carbon source for further scale-up production of *B. velezensis* BL-1 (Table 1). Many experiments have proven that in terms of the growth of *Bacillus subtilis*, sucrose was one of the optimal carbon sources in culture broths [15].

***Table 1**. Culture profile of Bacillus velezensis BL-1 using different carbon source*

Carbon source	Total viable cells (CFU/mL)
Glucose	3.37 x 10^8 ± 0.08 x 10^8
Glycerol	1.08 x 10^8 ± 0.02 x 10^8
Sucrose	4.24 x 10^8 ± 0.06 x 10^8
Lactose	2.23 x 10^8 ± 0.03 x 10^8

Note: Data present means ± standard error of the mean (SEM)

3.3.2. Identification of the optimal nitrogen source in batch cultivation

The effective bacterial growth in each of 3 nitrogen sources including Soybean flour, Yeast extract, and Peptone was tested by serial dilution and spreading *B. velezensis* BL-1 culture on plates for 24 hours. The results revealed that soybean flour was the most optimal nitrogen source which performed high efficiency (7.07 x 10^8 CFU/mL) in a low cost, and was suitable for mass production of *Bacillus velezensis* BL-1 (Table 2). The present result differed from the study of Peighmi-Ashnari et. al.(2009) which reported that molasses and yeast extract based media to be the most suitable for rapid growth and high cell yield of *B. subtilis* [9].

Table 2. *Culture profile of Bacillus velezensis BL-1 using different nitrogen sources*

Nitrogen source	Total viable cells (CFU/mL)
Soybean flour	7.07 x 10^8 ± 0.20 x 10^8
Yeast extract	3.41 x 10^8 ± 0.09 x 10^8
Peptone	4.51 x 10^7 ± 0.05 x 10^8

Note: Data present means ± standard error of the mean (SEM)

3.3.3. Identification of the optimal duration for batch fermentation

The optimal time length for batch fermentation of *B. velezensis* BL-1 was examined by performing bacteria cultrure for 2 days in medium of soybean flour (2%) flus molasses (10%). The result indicated that the viable cell reached the highest number at 48 hours of incubation (5.2 x 10^8 CFU/mL). This also suggested that the optimal fermentation conditions for *B. velezensis* BL-1 were 48 hours incubation at 35^{o}C, pH 7.0, and 2% of inoculum size (Table 3). Nakkeran et. al. (2006) reported that B. subtilis growth was good at temperature of 28±2^{o}C [7]. Whereas, Korsten and Cook (1996) reported that temperature of 30-37^{o}C and pH of 7-8 good for the growth and multiplication of *B. subtilis* [5].

Table 3. *Bacterial growth in the batch fermentation using soybean molasses-based medium*

Medium	Total viable cells (CFU/mL) after incubation (hours)		
	12	24	48
Soybean flour (2%) + Molasses (10%)	6.8 x10^7 ± 0.17 x 10^7	3.4 x 10^8 ± 0.28 x 10^8	5.2 x 10^8 ± 0.31 x 10^8

Note: Data present means ± standard error of the mean (SEM)

4. Conclusions

In the present study, a spore-forming and rod-shaped isolated BL-1 strain was successfully isolated from soil and identified as *Bacillus velezensis* based on morphological and phylogenetic analysis of 16S rRNA. We also proposed a protocol for *Bacillus velezensis* BL-1 mass production using soybean flour and molasses as an optimal nitrogen and carbon source based on batch culture fermentation technique. The optimal duration for batch fermentation of *Bacillus velezensis* BL-1 using medium of soybean flour (2%) plus molasses (10%) was a 48-hour incubation, which offered the highest yield of bacterial growth (> 5 $x10^8$ CFU/mL).

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FRÉCHET SINGULAR SUBDIFFERENTIALS OF THE MINIMAL TIME FUNCTION ASSOCIATED WITH A COLLECTION OF SETS

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Abstract: *In this paper, we present the formulas for computing the Fréchet singular subdifferentials of the minimal time function associated with a collection of subsets in normed spaces.*

Keywords: *Minimal time function, Fréchet singular subdifferentials, Normed spaces.*

1. Introduction and preliminaries

Let X be a normed space, F and Ω be two nonempty subsets of X. The minimal time function with the constant dynamics F and the target set Ω is defined by

$$T_{\Omega}^{F}(x) := \inf\{t \geq 0 : (x+tF) \cap \Omega \neq \emptyset\}, \quad x \in X. \tag{1.1}$$

The minimal time function T_{Ω}^{F} plays an important role in variational analysis since its covers three crucial functions in variational analysis: the distance function, the Minkowski function and the indicator function. Variational analysis and subdifferentials of the minimal time function with a convex dynamics containing the origin in its interior, in Hilbert spaces were first investigated by Colombo and Wolenski in [5] [6]. Later, the function has been studied extensively by many researchers; see, e.g., [2] [3] [4] [7] [8] [9] [12] [13] [16] [19] [21]. He [8] studied subdifferentials of the minimal time function in Banach spaces. These results were extended to the setting of normed spaces by Jiang and He in [9] and then improved by Mordukhovich and Nam in [12] [13]. Bounkhel investigated subdifferential calculus of the minimal time function in Hausdorff topological vector spaces in [2] [3]. Applications of variational analysis and generalized differentiations of the minimal time function to generalized location problems were presented in [13] [14] [15] [16] [17] [18] [20] and references therein.

The notion of the minimal time function associated with a collection of set was recently introduced in [11]. This new function contains the classical minimal time function as a special case. More precisely, let m be a positive integer and let $\mathcal{U} = \{U_1, \cdots, U_m\}$ be a collection of m nonempty subsets $U_1, \cdots, U_m$ of X and Ω a nonempty subset of X. The minimal time function associated with the collection $\mathcal{U}$ to the set Ω is defined as: for $x \in X$.

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$$T_{\mathcal{U},\Omega}(x) := \inf\left\{t_1+\cdots+t_m : t_1,\cdots,t_m \ge 0 \text{ and } \left(x+t_1U_1+\cdots+t_mU_m\right)\cap\Omega\ne\emptyset\right\}.$$

It is obvious that if $U_1 = F$ and $U_2 = \cdots = U_m = \{0\}$, then $T_{\mathcal{U},\Omega}$ becomes the usual minimal time function T_Ω^F defined in (1.1). Let $x \in X$. From the definition of the minimal time function $T_{\mathcal{U},\Omega}$, we see that if $T_{\mathcal{U},\Omega}(x) < \infty$, then it is the smallest time to steer x to the target Ω using at most one direction in each set $U_1,\cdots,U_m$. It means that x may be steered to the target Ω in a "zigzag" path. This contrasts with the case of the classical minimal time function as points are steered to the target along a straight path. It turns out that the new type of minimal time function is more flexible than the classical one and it can be used to model problems that the classical one cannot. By careful adaptation of existing results for the classical minimal time function, in [11], we present various basic properties of the new minimal time function. These properties (which include, among others, lower semicontinuity, Lipschitz continuity, convexity, principle of optimality and subdifferential calculus) were then utilized to study a location problem. The aim of this paper is to continue investigating the minimal time function associated with a collection of sets by proving the formulas for computing Fréchet singular subdifferentials of the function.

We now recall some basic concepts of nonsmooth analysis [1] [10]. Let X be normed space and X^* be the topological dual space of X. We denote by $\|\cdot\|$ the norm in X and by $\langle\cdot,\cdot\rangle$ the dual pair between X and X^*. We also denote by $\|\cdot\|$ the dual norm in X^*. Denote by $B(x,r)$ the open ball of radius $r>0$ centered at x and $\mathbb{B} = B(0,1)$.

Let $S \subset X$ be a closed set and let $x \in S$. The Fréchet normal cone to S at x, written $\hat{N}_S(x)$, is the set

$$\hat{N}_S(x) := \left\{\zeta \in X^* : \limsup_{S\ni y\to x} \frac{\langle \zeta, y-x\rangle}{\| y-x\|} \le 0\right\}.$$

In other words, $\zeta \in \hat{N}_S(x)$ if and only if for any $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\langle \zeta, y-x\rangle \le \varepsilon \| y-x\|, \qquad \forall y \in B(x,\delta).$$

Elements in $\hat{N}_S(x)$ are called Fréchet normals to S at x.

Let $f : X \to \mathbb{R}\cup\{+\infty\}$ be an extended real-valued function. The effective domain of f is defined by $\mathrm{dom}(f) := \{x \in X : f(x) < +\infty\}$ and the epigraph of f is defined by $\mathrm{epi}(f) := \{(x,\alpha) \in X\times\mathbb{R} : x \in \mathrm{dom}(f), \alpha \ge f(x)\}$. Let $x \in \mathrm{dom}(f)$. The Fréchet subdifferential of f at x is the set

$$\hat{\partial} f(x) := \left\{\zeta \in X^* : \liminf_{y\to x} \frac{f(y)-f(x)-\langle \zeta, y-x\rangle}{\| y-x\|} \ge 0\right\}.$$

Equivalently, $\zeta \in \hat{\partial} f(x)$ if and only if for any $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\langle \zeta, y-x\rangle \le f(y)-f(x)+\varepsilon\| y-x\|, \qquad \forall y \in B(x,\delta).$$

We call elements in $\hat{\partial} f(x)$ the Fréchet subgradients of f at x. The Fréchet subdifferential of f at x can also be defined as

$$\hat{\partial} f(x) = \left\{ \zeta \in X^* : (\zeta, -1) \in \hat{N}_{\mathrm{epi}(f)}(x, f(x)) \right\}.$$

The Fréchet singular subdifferential of f at x is the set $\hat{\partial}^\infty f(x)$ which is defined by

$$\hat{\partial}^\infty f(x) = \left\{ \zeta \in X^* : (\zeta, 0) \in \hat{N}_{\mathrm{epi}(f)}(x, f(x)) \right\}.$$

In other words, $\zeta \in \hat{\partial}^\infty f(x)$ if and only if for any $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\langle \zeta, y - x \rangle \le \varepsilon(\| y - x \| + | \beta - f(x) |), \qquad \forall y \in B(x, \delta), (y, \beta) \in \mathrm{epi}(f).$$

We call elements in $\hat{\partial}^\infty f(x)$ the Fréchet singular subgradients of f at x.

The support function $\rho_A : X^* \to (-\infty, \infty]$ of a subset A of X is defined as: for $\zeta \in X^*$

$$\rho_A(\zeta) = \sup_{x \in A} \langle \zeta, x \rangle.$$

2. Fréchet singular subdifferentials of the minimal time function

For simplicity of the presentation, we consider the minimal time function associated with a collection of two subsets of X. Throughout this section, $\mathcal{U} = \{U_1, U_2\}$ is a collection of two nonempty, bounded subsets U_1, U_2 of X. We always assume that $U_1 \cap U_2 \subset \{0\}$ and $U_1 \cup U_2 \ne \{0\}$, Ω is closed. We denote $M = \sup\{\| u \| : u \in \mathbb{U}\}$ where $\mathbb{U} = U_1 \cup U_2$. The function $T_{\mathcal{U},\Omega}$ is now written as:

$$T_{\mathcal{U},\Omega}(x) = \inf \left\{ t_1 + t_2 : t_1, t_2 \ge 0 \text{ and} \left(x + t_1 U_1 + t_2 U_2 \right) \cap \Omega \ne \varnothing \right\}. \tag{2.1}$$

For $t \ge 0$, we define

$$\mathcal{R}(t) := \{ x \in x : T_{\mathcal{U},\Omega}(x) \le t \},$$

and

$$\mathcal{R} := \{ x \in X : T_{\mathcal{U},\Omega}(x) < \infty \}.$$

Our first result is stated as follows.

Theorem 2.1. *Let* $x_0 \in \Omega$. *We have*

$$\hat{\partial}^\infty T_{\mathcal{U},\Omega}(x_0) = \hat{N}_\Omega(x_0) \cap \{ \zeta \in X^* : \max\{ \rho_{U_1}(-\zeta), \rho_{U_2}(-\zeta) \} \le 0 \}. \tag{2.2}$$

Proof. Let $\zeta \in \hat{\partial}^\infty T_{\mathcal{U},\Omega}(x_0)$. Then, for any $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\langle \zeta, y - x_0 \rangle \le \varepsilon(\| y - x_0 \| + \beta), \quad \forall y \in B(x_0, \delta), (y, \beta) \in \mathrm{epi}(T_{\mathcal{U},\Omega}). \tag{2.3}$$

It follows that

$$\langle \zeta, y - x_0 \rangle \le \varepsilon \| y - x_0 \|, \quad \forall y \in \Omega \cap B(x_0, \delta).$$

This means that $\zeta \in \hat{N}_\Omega(x_0)$.

Let $u \in \mathbb{U}$ be arbitrary and let $\lambda > 0$ be sufficiently small such that $y := x_0 - \lambda u \in B(x_0, \delta)$. Then, we have $T_{\mathcal{U},\Omega}(y) \le \lambda$. From (2.3), one has

$$\langle \zeta, -\lambda u \rangle \le \varepsilon(\| -\lambda u \| + \lambda)$$

Dividing both sides of the latter inequality by $\lambda > 0$, we get

$$\langle \zeta, -u \rangle \le \varepsilon(\| u \| + 1).$$

Letting $\varepsilon \to 0^+$, we have $\langle \zeta, -u \rangle \le 0$. Since $u \in \mathbb{U}$ is arbitrary, $\rho_{\mathbb{U}}(-\zeta) \le 0$, or

$$\max\{\rho_{U_1}(-\zeta), \rho_{U_2}(-\zeta)\} \le 0.$$

Now, let $\zeta \in \hat{N}_\Omega(x_0)$ be such that

$$\max\{\rho_{U_1}(-\zeta), \rho_{U_2}(-\zeta)\} \le 0.$$

We shall prove that $\zeta \in \hat{\partial}^\infty T_{\mathcal{U},\Omega}(x_0)$. Assume to the contrary that $\zeta \notin \hat{\partial}^\infty T_{\mathcal{U},\Omega}(x_0)$ Then, there exist $C > 0$ and sequences $\{y_i\} \subset X$, $\{\beta_i\} \subset \mathbb{R}$ such that $y_i \to x_0$ as $i \to \infty$, and $\beta_i \ge T_{\mathcal{U},\Omega}(y_i), y_i \ne x_0$ and

$$\langle \zeta, y_i - x_0 \rangle \ge C(\| y_i - x_0 \| + \beta_i). \tag{2.4}$$

for all i. We have from (2.4) that

$$\langle \zeta, y_i - x_0 \rangle \ge C(\| y_i - x_0 \| + T_{\mathcal{U},\Omega}(y_i)), \qquad \forall i. \tag{2.5}$$

This yields

$$t_i := T_{\mathcal{U},\Omega}(y_i) \le \frac{1}{C} \| \zeta \| \| y_i - x_0 \|, \quad \forall i$$

and thus $t_i \to 0$ as $i \to \infty$.

Let $\eta > 0$. By the definition of $T_{\mathcal{U},\Omega}$, for each i, there exist $t_1^i, t_2^i \ge 0$, $w^i \in \Omega$, $u_1^i \in U_1$, $u_2^i \in U_2$ such that

$$t_i < t_1^i + t_2^i < t_i + \eta, \quad w^i = y_i + t_1^i u_1^i + t_2^i u_2^i$$

One has for all i that

$$\| w^i - x_0 \| = \| y_i + t_1^i u_1^i + t_2^i u_2^i - x_0 \| \le \| y_i - x_0 \| + (t_i + \eta) M.$$

Since $\eta > 0$ is arbitrary, $w^i \to x_0$ as $i \to \infty$. Let $\varepsilon > 0$. Since $\zeta \in \hat{N}_\Omega(x_0)$, for i large enough, we have

$$\langle \zeta, w^i - x_0 \rangle \le \varepsilon \| w^i - x_0 \|, \tag{2.6}$$

For i large enough,

$$\begin{aligned} C(\| y_i - x_0 \| + t_i) &\le \langle \zeta, y_i - x_0 \rangle = \langle \zeta, w^i - t_1^i u_1^i - t_2^i u_2^i - x_0 \rangle \\ &= \langle \zeta, w^i - x_0 \rangle + t_1^i \langle -\zeta, u_1^i \rangle + t_2^i \langle -\zeta, u_2^i \rangle \\ &\le \varepsilon \| w^i - x_0 \| \quad (\text{as } \rho_{U_i}(-\zeta) \le 0). \end{aligned}$$

Hence,

$$C \le \varepsilon \frac{\| w^i - x_0 \|}{\| y_i - x_0 \| + t_i}, \tag{2.7}$$

for i large enough. We claim that the sequence $\left\{ \frac{\| w^i - x_0 \|}{\| y_i - x_0 \| + t_i} \right\}$ is bounded. Assume to the contrary that the sequence is not bounded. Then, without loss of generality, there exists i_0 such that for $i > i_0$, we have

$$\frac{\| w^i - x_0 \|}{\| y_i - x_0 \| + t_i} > M + 2. \tag{2.8}$$

That is, for $i > i_0$,

$$(M+2)(\| y_i - x_0 \| + t_i) \le \| w^i - x_0 \| \le \| y_i - x_0 \| + (t_i + \varepsilon)M$$

Let $\varepsilon \to 0^+$, one has

$$(M+2)(\| y_i - x_0 \| + t_i) \le \| y_i - x_0 \| + t_i M$$

for all i sufficiently large. This implies that

$$(M+1)\| y_i - x_0 \| \le -2t_i < 0$$

for all $i > i_0$ large enough. This is a contradiction.

Set

$$Q = \sup_i \left\{ \frac{\| w^i - x_0 \|}{\| y_i - x_0 \| + t_i} \right\}.$$

From (2.7), we have $C \le \varepsilon Q$. Let $\varepsilon \to 0^+$, one gets $C \le 0$ which is a contradiction. Thus, $\zeta \in \hat{\partial}^\infty T_{\mathcal{U},\Omega}(x_0)$. This ends the proof.

Next, we give the formula for computing Fréchet singular subdifferentials of the minimal time function at a point outside the target. For that aim, we need the following result.

Proposition 2.1. *Let* $x_0 \in X$ *be such that* $0 < r := T_{\mathcal{U},\Omega}(x_0) < +\infty$. *If* $\zeta \in \hat{N}_{\mathcal{R}(r)}(x_0)$ *then we have* $\rho_{\mathbb{U}}(-\zeta) \ge 0$.

Proof. Since $\zeta \in \hat{N}_{\mathcal{R}(r)}(x_0)$, for any $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\langle \zeta, y - x_0 \rangle \le \varepsilon \| y - x_0 \|. \tag{2.9}$$

for all $y \in B(x_0, \delta) \cap \mathcal{R}(r)$.

Since $r = T_{\mathcal{U},\Omega}(x_0) < +\infty$, by the definition of $T_{\mathcal{U},\Omega}$, for $0 < \gamma < r/2$, there exist $t_1, t_2 \ge 0$, $w \in \Omega$, $u_1 \in U_1$, $u_2 \in U_2$ such that $r < t_1 + t_2 < r + \gamma$ and $w = x_0 + t_1 u_1 + t_2 u_2$. Without loss of generality, we may assume that $t_1 \ge t_2$ and $t_1 > 0$. Then, $t_1 > r/2$.

We take $r/2 < \eta < \max\{r, \delta/M\}$ and let $z = x_0 + \eta u_1$. It is easy to see that $z \in B(x_0, \delta)$. Moreover, since

$$z + (t_1 - \eta)u_1 + t_2 u_2 = w \in \Omega,$$

we have

$$T_{\mathcal{U},\Omega}(z) \le t_1 - \eta + t_2 < r + \gamma - \eta \le r.$$

It means that $z \in B(x_0, \delta) \cap \mathcal{R}(r)$. From (2.9), one has $\langle \zeta, \eta u_1 \rangle \le \varepsilon \| \eta u_1 \|$. Equivalently,

$$\langle \zeta, u_1 \rangle \le \varepsilon \| u_1 \|.$$

Let $\varepsilon \to 0+$, we get $\langle \zeta, u_1 \rangle \le 0$. Therefore, $\rho_U(-\zeta) \ge 0$. This ends the proof.

Theorem 2.2. *Let U_1 and U_2 be convex and $x_0 \in X$ such that $0 < r := T_{\mathcal{U},\Omega}(x_0) < \infty$. Then,*

$$\hat{\partial}^\infty T_{\mathcal{U},\Omega}(x_0) = \hat{N}_{\mathcal{R}(r)}(x_0) \cap \{\zeta \in X^* : \max\{\rho_{U_1}(-\zeta), \rho_{U_2}(-\zeta)\} = 0\}. \tag{2.10}$$

Proof. Assume that $\zeta \in \hat{\partial}^\infty T_{\mathcal{U},\Omega}$. Then, for any $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\langle \zeta, y - x_0 \rangle \le \varepsilon(\| y - x_0 \| + | \beta - r |), \quad \forall y \in B(x_0, \delta), (y, \beta) \in \operatorname{epi}(T_{\mathcal{U},\Omega}). \tag{2.11}$$

It follows that $\langle \zeta, y - x_0 \rangle \le \varepsilon \| y - x_0 \|, \forall y \in \mathcal{R}(r) \cap B(x_0, \delta),$ that is, $\zeta \in \hat{N}_{\mathcal{R}(r)}(x_0)$.

Since $r = T_{\mathcal{U},\Omega}(x_0) < +\infty$, for $0 < \gamma < r^2/4$, there exist $t_1, t_2 \ge 0$, $w \in \Omega$, $u_1 \in U_1$, $u_2 \in U_2$ such that $r < t_1 + t_2 < r + \gamma, \quad w = x_0 + t_1 u_1 + t_2 u_2$.

Let $u \in \mathbb{U}$ be arbitrary and $\lambda > 0$. Assume that $u \in U_1$. Then, by the convexity of U_1, one has $w \in x_0 - \lambda u + \lambda U_1 + t_1 U_1 + t_2 U_2 = x_0 - \lambda u + (t_1 + \lambda)U_1 + t_2 U_2$.

Thus, $T_{\mathcal{U},\Omega}(x_0 - \lambda u) \le t_1 + t_2 + \lambda < r + \gamma + \lambda$. For λ sufficiently small, we have $x_0 - \lambda u \in B(x_0, \delta)$. By (2.11), one has $\langle \zeta, -\lambda u \rangle \le \varepsilon(\| -\lambda u \| + | \lambda + \gamma |)$.

Let $\gamma \to 0^+$, we get $\lambda \langle -\zeta, u \rangle \le \lambda \varepsilon (\| u \| + 1)$.

Divide both sides of the latter inequality by $\lambda > 0$ and then let $\varepsilon \to 0^+$, we obtain $\langle -\zeta, u \rangle \le 0$. Similarly, if $u \in U_2$, we can also show that $\langle -\zeta, u \rangle \le 0$. Since $u \in \mathcal{U}$ is arbitrary, $\rho_{\mathbb{U}}(-\zeta) \le 0$. Combining with Proposition 2.1, we have $\rho_{\mathbb{U}}(-\zeta) = 0$.

Conversely, let $\zeta \in \hat{N}_{\mathcal{R}(r)}(x_0)$ be such that $\rho_{\mathbb{U}}(-\zeta) = 0$. We show that $\zeta \in \hat{\partial}^\infty T_{\mathcal{U},\Omega}(x_0)$. Assume to the contrary that $\zeta \notin \hat{\partial}^\infty T_{\mathcal{U},\Omega}(x_0)$. Then, there exists $\alpha > 0$ and sequences $\{y_i\} \subset X$, $\{\beta_i\} \subset \mathbb{R}$ such that $y_i \to x_0$, $y_i \ne x_0$, $r_i := T_{\mathcal{U},\Omega}(y_i) \le \beta_i$ and\

$$\langle \zeta, y_i - x_0 \rangle \ge \alpha(\| y_i - x_0 \| + | \beta_i - r |), \quad \forall i. \tag{2.12}$$

We consider two cases:

Case 1. There exists a subsequence of $\{y_i\}$ which we still denote by $\{y_i\}$ such that $T_{\mathcal{U},\Omega}(y_i) \le T_{\mathcal{U},\Omega}(x_0)$ for all i. In this case, $y_i \in \mathcal{R}(r)$ for all i. Since $\zeta \in \hat{N}_{\mathcal{R}(r)}(x_0)$, for any $\varepsilon > 0$,

$$\langle \zeta, y_i - x_0 \rangle \le \varepsilon \| y_i - x_0 \|$$

for i large enough. Combining with (2.12), one gets

$$\varepsilon \| y_i - x_0 \| \ge \alpha(\| y_i - x_0 \| + | \beta_i - r |) \ge \alpha \| y_i - x_0 \|$$

which implies $\alpha \le \varepsilon$. This is a contradiction.

Case 2. There exists a subsequence of $\{y_i\}$ which is still denoted by $\{y_i\}$ such that $T_{\mathcal{U},\Omega}(y_i) > T_{\mathcal{U},\Omega}(x_0)$ for all i. By (2.12), $r_i = T_{\mathcal{U},\Omega}(y_i) < +\infty$ and

$$0 < r_i - r \le \beta_i - r \le \frac{1}{\alpha} \| \zeta \| \| y_i - x_0 \|$$

for all i. It implies that $r_i \to r$ as $i \to \infty$. Hence, we may assume that $5(r_i - r) < r$ for all i. By the definition of the minimal time function, for each i, there exist $t_1^i, t_2^i \ge 0$, $w_i \in \Omega$, $u_1^i \in U_1$, $u_2^i \in U_2$ such that

$$r_i < t^i - 1 + t_2^i < 2r_i - r, \quad w_i = y_i + t_1^i u_1^i + t_2^i u_2^i.$$

Without loss of generality, we may assume that $t_1^i \ge t_2^i$. Then, for all i,

$$t_1^i \ge \frac{1}{2}(t_1^i + t_2^i) > \frac{1}{2} r_i > 3(r_i - r).$$

For each i, let $\gamma_i \in (2r_i - 2r, 3r_i - 3r)$. Then,

$$w_i = y_i + \gamma_i u_i^i + (t_1^i - \gamma_i) u_1^i + t_2^i u_2^i.$$

Thus, $T_{\mathcal{U},\Omega}(y_i + \gamma_i u_1^i) \le t_1^i - \gamma_i + t_2^i < 2r_i - r - (2r_i - 2r) = r.$

This means that $y_i + \gamma_i u_1^i \in \mathcal{R}(r)$. Moreover,

$$\| y_i + \gamma_1^i u_i - x_0 \| \le \| y_i - x_0 \| + 3(r_i - r)M \to 0 \quad \text{as}\, i \to \infty.$$

That is, for i large enough, $y_i + \gamma_1^i u_i \in \mathcal{R}(r) \cap B(x_0, \delta)$. Let $\varepsilon > 0$. Since $\zeta \in \hat{N}_{\mathcal{R}(r)}(x_0)$, for i sufficiently large,

$$\langle \zeta, y_i + \gamma_1^i u_i - x_0 \rangle \le \varepsilon \| y_i + \gamma_1^i u_i - x_0 \|.$$

Since $\rho_{\mathbb{U}}(-\zeta) = 0$,

$$\langle \zeta, y_i - x_0 \rangle \le \varepsilon \| y_i + \gamma_i u_1^i - x_0 \| + \gamma_i \langle -\zeta, u_1^i \rangle \le \varepsilon \| y_i + \gamma_i u_1^i - x_0 \|.$$

Combining with (2.12), one has

$$\alpha(\| y_i - x_0 \| + | r_i - r |) \le \alpha(\| y_i - x_0 \| + | \beta_i - r |) \le \varepsilon \| y_i + \gamma_i u_1^i - x_0 \|.$$

Thus,

$$\begin{aligned}\alpha &\le \varepsilon \frac{\| y_i + \gamma_i u_1^i - x_0 \|}{\| y_i - x_0 \| + | r_i - r |} \\ &\le \varepsilon \frac{\| y_i - x_0 \| + 3M | r_i - r |}{\| y_i - x_0 \| + | r_i - r |} \\ &\le (3M+1)\varepsilon.\end{aligned}$$

Letting $\varepsilon \to 0^+$, we have $\alpha \le 0$. This is a contradiction. Therefore, $\zeta \in \hat{\partial}^\infty T_{\mathcal{U},\Omega}(x_0)$. The proof is complete.

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CHEMICAL COMPOSITION, ANTIMICROBIAL AND ANTIOXIDANT ACTIVITITES OF ESSENTIAL OIL FROM (*OCIMUM BASILICUM* L.) COLLECTED IN THANH HOA PROVINCE

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Abstract: *The leaves of Ocimum basilicum* L. *were collected in Thanh Hoa province. Chemical composition of (Ocimum basilicum* L.*) essential oil has been examined by GC-MS. The identified components constitute 0.5-0.6%. There are 23 compounds and the main compounds are estragole (92.16%), 1.8-Cineole (1,69%), 3-Carene (0.99%), and trans-α-Bergamotene (0.52%). Estragole (92.16%) was higher than the previously published. In addition, we also tested the antibacterial activity of (Ocimum basilicum* L.*) essential oil on* E. coli*. The results showed that the essential oil had strong antibacterial activity of about 89.20% on* E. coli*. The antioxidant capacity of basil essential oil was determined by DPPH method. The highest antioxidant efficiency of basil essential oil was 96.40% (IC_{50} = 1,08 µg/mL) at 2.5 mg/mL. The results of this research show that the of Ocimum basilicum* L. *might be a natural potential source of antibacterial and antioxidants.*

Keywords: *Ocimum basilicum L, antibacterial, E.coli, essentialoil, antioxidant.*

1. Introduction

Ocimum basilicum L. is also known as basil and purple, which is grown very popularly in our country to make spices. Basil has a warm spicy taste, so it is used in medicine to treat colds, flu, indigestion, etc. In addition, basil contains 0.4-0.8% essential oils [1]. Since 1975, a number of provinces have grown on a large scale to store essential oils for domestic and foreign aromatics industry. According to the report of author Tran Thanh Quynh Anh and colleagues [2], analysis by gas chromatography coupled mass spectrometry (GC-MS) showed that basil essential oil (*Ocimum basilicum* L.) in Thua Thien Hue has the main chemical components such as: p-allylanisole (49.09%), aromadendrene (8.27%) and trans-ocimene (5.71%). The study also investigated the antioxidant and antibacterial activities of essential oils. The results indicated that basil essential oil had low antioxidant capacity (IC_{50}=35.89 µg/mL) and was able to inhibit two strains of *E. coli*, *Salmonellasp*. There have been studies showing that materials grown in different geographical locations and soils lead to different chemical compositions and biological activities of essential oils. However, there is still no research on the chemical

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composition and biological activity of basil essential oil in Thanh Hoa. Therefore, we conducted this study to create a database of *Ocimum basilicum* L. in Thanh Hoa, thereby contributing to the efficient exploitation and use of the species.

Figure 1. *Ocimum basilicum* L. - *Thanh Hoa*

2. Experiment

2.1. Equipment: Light Clevenger Distillation Kit, Thermo Trace GC Ultra GC-MS analyzer system- ITQ900.

2.2. Chemistry: Anhydrous Na_2SO_4, diethyl ether.

2.3. Material

Ocimum basilicum L. was collected in Quang Xuong district, Thanh Hoa province, Vietnam in June 2021.

Parts Used: Leaves

2.4. Method

2.4.1. Distillation method

After collection, samples were preliminarily selected, washed and dried for 2 hours, cut into small pieces. Place in the Clevenger distillation flask 150 g chopped basil leaves with 500 ml of water. Distill for 2.5 hours, the mixture is heated by an electric stove, when the mixture boils, the steam formed will attract the essential oil up and into the condenser system. After condensing to obtain an insoluble mixture of water and essential oils, extracting the essential oil from the mixture with diethyl ether, anhydrous the extracted solution with anhydrous Na_2SO_4 salt, and obtain the finished essential oil. The yield of essential oil was 0.6 %. The oil is light yellow in color and has a mild fragrance and a density of 0.953 g/mL

2.4.2. Method for determining chemical composition

The chemical composition of essential oils was determined by gas chromatography-mass spectrometry (GC-MS), measured at the Institute of Biotechnology and Environment, Tay Nguyen University. Using Thermo Trace GC Ultra - ITQ900 GC/MS machine, TG-SQC chromatographic column with the length of 30 m, inner diameter (ID) = 0.25 mm, thin film of 0.25 nm. Heli carrier gas. Sample injection chamber temperature (Temperature Program Technique-PTV) of 250^{o}C, Detector temperature of 260^{o}C. The thermostatic chamber temperature program: 60^{o}C (2min), increasing 4^{o}C/min until 200^{o}C, stopping at this temperature for 5 minutes, increasing 10^{o}C/min until 260^{o}C, stopping at this temperature for 10 minutes.

2.4.3. Test method for antibacterial activity

Antibacterial activity was tested at the Institute of Biotechnology and Environment, Tay Nguyen University, by using diffusion on agar plate with MHA medium.

Tested bacteria strain: *Escherichia coli*. Active test steps:

Preparation of test microorganisms: strains before use are grown on TSB medium for 16-18 hours at 37^{o}C, shaken at 100 rpm. Bacterial density after culture in TSB medium was determined by optical densitometry (OD) at 610 nm.

Prepare essential oil solution: essential oil is dissolved in DMSO 2%, using emulsifier is Tween 80 - 0.2%. The control solution consisted of 2% DMSO, using 0.2% emulsifier tween 80 in distilled water.

Use a pipette to suck up 100 µl of bacteria (cell density 10^8 CFU/ml), then spread evenly on the surface of the stable dried MHA agar, wait for the surface to dry. Use sterile 6mm paper plates saturated with essential oil solutions at different concentrations and control solutions, wait to dry and then place on the infested agar surface, gently press the paper plate to fix on the agar surface. Transfer the petri dishes to the refrigerator (10^{o}C) for about 4 - 8 hours for the essential oils to diffuse into the agar. Then rear at 37^{o}C for 16 - 20 hours. Read the results and record the diameter of the sterile ring.

The diameter of the sterile ring (D-d) is determined by the diameter of the outer ring minus the diameter of the paper plate.

2.4.4. Antioxidant activity test

Antioxidant activity of basil essential oil was carried out at the Institute of Biotechnology and Environment of Tay Nguyen University. The test process is carried out according to the DPPH free radical scavenging method: A method to determine the antioxidant capacity of a compound based on its free radical scavenging ability. DPPH (1,1-diphenyl-2-picrylhydrazyl) is capable of generating stable free radicals in methanol and has a maximum absorbance at 517 nm. When the test samples are added to this mixture, if the substance has the ability to neutralize or encapsulate the free radicals, the DPPH will change from purple to yellow. This signal is measured with an ELISA reader. The antioxidant activity of the test substance was evaluated through the percentage reduction in the light absorption value of the test sample compared to the control. The antioxidant activity test results are reported by the IC50 value as the concentration of the extract that can reduce 50% of DPPH free radicals under specified conditions. The lower the IC50 value is the higher the DPPH free radical scavenging activity becomes. The inhibition of DPPH was calculated according to the following formula:

$$\% \text{ IC} = \frac{\text{OD}_{froaf} - \text{OD}_{\text{try on}}}{\text{OD}_{\text{froaf}} - \text{OD}_{\text{white}}} \text{ x } 100\%$$

Where: OD: absorbance of control sample (no sample);

Test OD: absorbance of sample;

OD blank: absorbance of the blank (methanol).

The IC50 value of the test sample and the control sample is based on a linear equation between their concentration and % free radical scavenging activity, calculated by the formula: IC50= (50-b)/a.

In there: IC50: is the concentration of the sample that can capture 50% of the free radical DPPH

a, b are the slope and intercept of the linear equation between concentration and % free radical capture, respectively.

3. Results and Discussion

3.1. Chemical composition results

The essential oil of *Ocimum basilicum* L. obtained is light yellow in color and has a mild fragrance. The GC-MS chromatogram of basil essential oil is shown in Figure 2. The chemical composition of the essential oil is shown in Table 1.

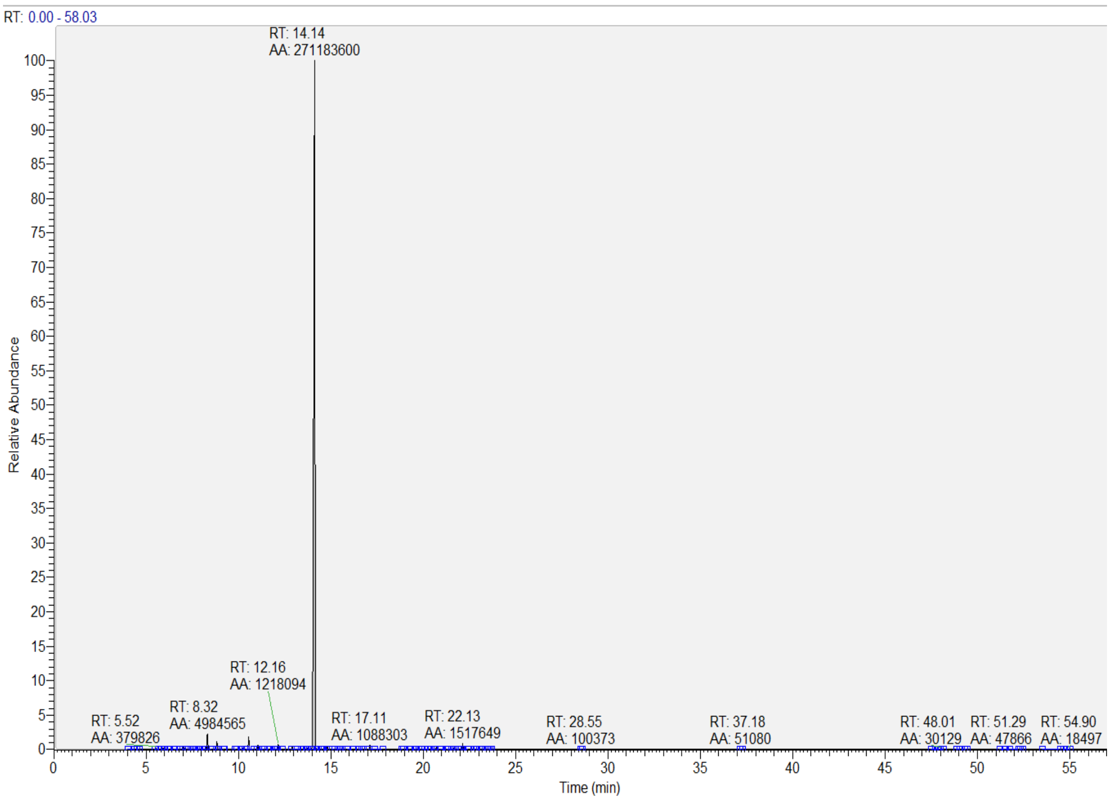
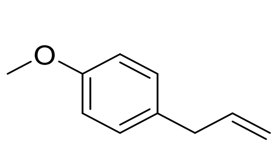
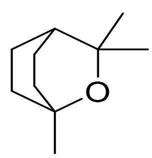


Figure 2. *GC-MS chromatogram of essential oil Ocimum basilicum L.*

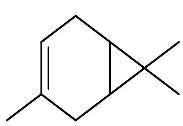
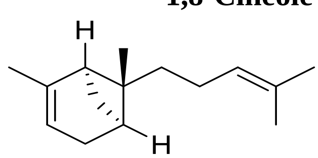
Table 1. *Chemical composition of essential oil Ocimum basilicum* L.

TT	**RT**	Compounds	**kind**	**AI^r**	**Area(%)**
1	5.52	α-Thujene	mh	**924**	0,13
2	5.92	β-Phellandrene	mh	1025	0,09
3	6.69	Sabinene	mh	969	0,17
4	7.04	β-Terpinene	mh	-	0,28
5	8.09	*p*-Cymene	mh	1020	0,07
6	**8.32**	**1,8-Cineole**	**mh**	**1026**	**1,69**
7	8.83	γ-Terpinene	mh	1054	0,62
8	10.21	3-ρ-Menthen-7-al	mo	-	0,17
9	**10.55**	**3-Carene**	**mh**	**-**	**0,99**
10	11.06	2-β-Pinene	mh	974	0,36
11	12.16	(+)-2-Bornanone	mh	1141	0,41
12	12.91	Camphene	mh	946	0,27
13	13.31	ι-Phellandrene	mh	-	0,07
14	13.78	δ-3-Carene	mh	1008	0,22
15	**14.14**	**Estragole**	**mh**	**1195**	**92,16**
16	14.37	ρ-Mentha-1,8-dien-7-ol	mo	1003	0,06
17	14.81	Z-Citral	mo	1235	0,23
18	16	unknow	-	-	0,05
19	17.11	Endobornyl acetate	mo	-	0,37
20	20.19	α-Cubebene	sh	1348	0,05
21	20.72	(-)-Sinularene	sh	-	0,17
22	21.12	cis-Methyl isoeugenol	mo	1451	0,2
23	21.65	trans-Caryophyllene	sh	1417	0,12
24	**22.13**	**trans-α-Bergamotene**	**mh**	**1432**	**0,52**
25	-	Other compounds	-	-	0,53
Total					100
Hydrocarbons					98,39
Derivatives containing oxyge					1,03

Chemical structures of the main compounds:

Estragole **1,8-Cineole**

3-Carene **trans-α-Bergamotene**

mh: monoterpene hydrocarbons; mo: oxygenated monoterpenes; sh: sesquiterpene hydrocarbons; nt: non-terpenes; AI^r: Reported Arithmetic Index.

Through analysis, the chemical composition of *Ocimum basilicum* L.essential oil has 23 identified components (accounting for 99.42%). Of which, hydrocarbons accounted for 98.39% and oxygen derivatives accounted for 1.03%. The main components in the essential oil are: Estragole (92.16%), 1,8-Cineole (1.69%), 3-Carene (0.99%) and trans-α-Bergamotene (0.52%). Compared with the national standard TCVN 11887:2017, the Estragole content is higher, the 1,8-Cineole content is similar. This proves that the quality of basil essential oil obtained is of good quality.

The results of our study on the chemical composition of essential oil of *Ocimum basilicum* L. collected in Thanh Hoa have many similarities with those published by Vo Thi Thanh Tuyen et al [3]. Research has found 25 constituents in essential oils extracted from plants grown in Quy Nhon City, Binh Dinh Province. The main components of the essential oil include: Estragole (85.92%), (E)-β-ocimen (1.95%), 1,8-Cineole (1.66%) and trans-α-Bergamotene (1.65%). The two components Estragole and 1,8-Cineole in our study have higher concentrations. However, the trans-α-Bergamotene conjugate was found in lower concentrations. The conjugate (E)-β-ocimen in this report was not found in our study. Another report by author Thien Hien Tran et. al. [4], also indicated that the main component of essential oil *Ocimum basilicum* L. collected in Ho Chi Minh City is Estragole (87.90%). Compared with the estragole content in essential oil collected in Thanh Hoa (92.16%), it is lower.

More interestingly, when comparing the research results by author Tran Thanh Quynh Anh et. al. It is shown that the essential oil content of *Ocimum basilicum* L. collected in Thua Thien Hue province has some compounds. The main components such as p- Estragole (49.09%) were lower than that of Estragole (92.16%) of in Thanh Hoa and the remaining main components Aromadendrene (8.27%), trans-Ocimene (5.71%) was completely absent in our study [2].

Along with the domestic studies, foreign studies were also reported. Accordingly, the authors Chalchat, J.-C and colleagues [5] investigated the chemical composition of essential oils from the leaves of *Ocimum basilicum* L. species in Mersin province. The results showed that essential oils have the main components: Estragole (58.60%), Limonene (13.54%), Exo-fenchyle acetate (10.99%) and Fenchone (5.7%). Compared with our study and the two studies mentioned above, the estragole content in this study was much lower. Notably, the remaining components were not present in our study. In addition, many other constituents were also found in both studies: α-Thujene, Sabinene, Sabinene and p-Cymene.

This result is higher than the study of Rajesh et. al. (2014) with estragole accounting for 38.3% and much lower estragole content in essential oil of *Ocimum basilicum* L. collected in Thanh Hoa. (92.16%). In addition, the content of 1.8-Cineole accounting for 1.66% in the essential oil from *Ocimum basilicum* L.in Thanh Hoa was much lower than the study of Ismail (2006) when researching essential oil. cinnamon in India with 1.8-Cineole content of (13.65%) [6].

Another study by author Al Abbasy, D. W and colleagues [1] showed that the main chemical composition of the essential oil of Omen species *(Ocimum basilicum* L.) is: linalool (69.87%), geraniol (9.75%), p-allylanisole (6.02%), 1.8-cineole (4.90%), trans-α-bergamotene (2.36%) and neryl acetate (1.24%)). The components linalool, geraniol, p-allylanisole and neryl acetate were not present in our study. However, two components, 1.8-cineole and trans-α-bergamotene, appeared in our study but at higher concentrations than in our study. The above analysis results show that the composition and chemical content of essential oil *Ocimum basilicum* L. is different depending on factors of geography, climate and soil conditions.

3.2. Results of the assessment of biological activity

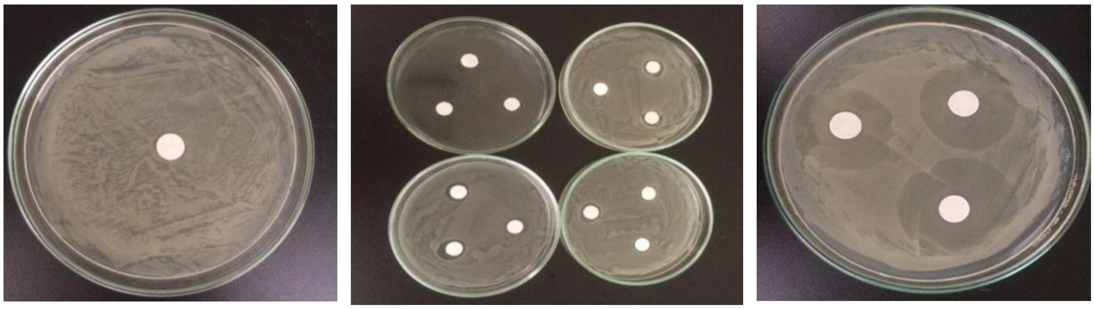
The antibacterial activity of the essential oil of *Ocimum basilicum* L. was determined based on its ability to inhibit the growth of bacteria, shown by the diameter of the antibacterial-halo zonering. The results of antibacterial activity evaluation are presented in Tables 2 and 3 below:

Table 2. *Positive test Ampicillin*

Bacterial density	**Essentialoil concentration (mg/ml)**	**Diameter of sterile ring (mm)**	**Resistance**
10^7 CFU	2,00	54mm, 54mm, 54mm	100% (b*acteria* do not grow all over the agar plate)
10^7 CFU	1,75	54mm, 54mm, 54mm	100% (*bacteria* do not grow all over the agar plate)
10^7 CFU	1,50	54mm, 54mm, 54mm	100% (*bacteria* do not grow all over the agar plate)
10^7 CFU	1,00	54mm, 54mm, 54mm	100% (bacteria do not grow all over the agar plate)
10^7 CFU	0,75	52mm, 52.5mm, 52.5mm	96.91%
10^7 CFU	0,50	50.5mm, 50mm, 50mm	92.90%

Table 3. *Antibacterial activity of essential oil Ocimum basilicum L.*

Bacterial density	**Essentialoil concentration (mg/ml)**	**Diameter of sterile ring (mm)**	**Resistance**
10^7 CFU	14	54mm, 54mm, 54mm	100% (bacteria do not grow all over the agar plate)
10^7 CFU	12	48 mm, 48mm, 48.5mm	89.20%
10^7 CFU	10	41.5mm, 41.5mm, 41mm	76.54%
10^7 CFU	8	32.5 mm, 32.5mm, 32.5mm	60.19%
10^7 CFU	6	24.5mm, 24mm, 24mm	44.75%
10^7 CFU	4	14mm; 14mm, 14.5mm	26.23%
10^7 CFU	2	6.5mm, 6mm, 6mm	11.42%



control (-) (DMSO 2%)

Figure 3. *Antibacterial ability at different concentrations of essential oil*

With a dilution concentration of 2.0 mg/ml, the antibacterial circle with the diameter measured against the resistance ring of *E. coli* is 6.2 mm, the inhibitory ability is 11.42%. The ability to inhibit *E. coli* bacteria increased gradually from the concentration of 2.0 mg/ml to 12.0 mg/ml and to the concentration of 14 mg/ml, the bacteria did not grow on the whole plate. At the concentration of 12.0 mg/ml, the antibacterial circle was 48.2 mm and the ability to inhibit *E. coli* was 89.20%. Compared with the study of author Tran Thanh Quynh Anh and colleagues, the antibacterial ability of Basil essential oil (*Ocimum basilicum* L.) in Hue City (*E. coli* antibacterial circle is 5.08 mm) and author Vo Thi Thanh Tuyen in Binh Dinh (16.0 mm *E.coli* antibacterial circle), the results showed the ability of basil essential oil to inhibit *E. coli* bacteria (*Ocimum basilicum* L.) in Thanh Hoa is stronger [2] [3]. This result is consistent with the study by Lu et al (2011) that basil essential oil is a potential natural antibacterial agent. Therefore, the combination of basil essential oil with other essential oils will increase its antibacterial ability. The results of antioxidant activity evaluation are presented in Tables 4 and 5 below:

Table 4. *Positive test: Acid ascorbic*

Acid ascorbic	**% Inhibition**			IC_{50} (µg /ml)
µg/ml	1	2	3	
50	70.87	70.85	70.89	34.99
40	56.11	56.02	56.14	
30	41.95	41.98	42.09	
20	30.17	30.14	30.07	
10	18.17	18.09	18.16	

Table 5. *Antioxidant activity of essential oil Ocimum basilicum L.*

Essential oil	**% Inhibition**			IC_{50} (mg/ml)
mg/ml	1	2	3	
2.5	96.93	96.11	96.16	1.08
2.0	80.28	80.76	80,45	
1.5	62.24	62.15	62,17	
1.0	47.15	47.18	47,23	
0.5	32.17	31.98	32,06	
0.25	22.76	22.78	22,86	

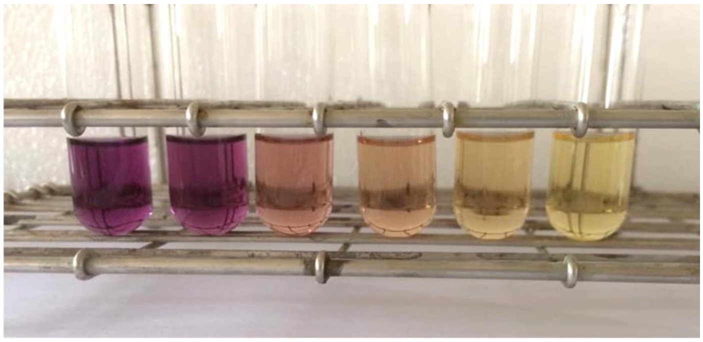


Figure 4. *Antioxidant capacity at different concentrations of essential oil*

The test showed that when the concentration of essential oil was from 0.25 mg/mL to 2.5 mg/mL, the percentage of DPPH free radical inhibitory activity of basil essential oil (*Ocimum basilicum* L.) gradually increased from 22, 80% to 96.40% (Table 5). The best antioxidant activity was demonstrated in sample 6 with the percentage of DPPH free radical scavenging activity of 94.40%. In addition, compared with the positive control ascorbic acid with IC50 = 34.99 µg/mL while basil essential oil with IC50 = 1.08 µg/mL, it can be concluded that the antioxidant capacity of essential oil is 32.5 times stronger than ascorbic acid. Compared with the study of author Tran Thanh Quynh Anh and colleagues, the antioxidant capacity of essential oil *Ocimum basilicum* L. in Hue City with IC50 = 35.89 µg/mL, the antioxidant capacity of essential oil of *Ocimum basilicum* L. in Thanh Hoa is much stronger [2].

This result also shows that the antioxidant activity of *Ocimum basilicum* L.essential oil in Thanh Hoa is much higher than that of Hadj-Khelifa et al. (2012), essential oil. *O. basilicum* from Algeria exhibited lower antioxidant activity (IC50 = 83.4 mg/mL) [7].

Comment: Through the test table, it was found that basil essential oil has a very strong antioxidant capacity. With IC50 = 1.08 mg/ml at a concentration of 0.25-2.5 mg/ml compared to the control substance Ascorbic Acid (IC50 (mg/ml) = 34.99) is 32.5 times higher. This result opens up the potential for the antioxidant capacity of *Ocimum basilicum* L.in the future.

4. Conclusion and Proposal

The essential oil of *Ocimum basilicum* L. was collected in Quang Xuong district, Thanh Hoa province. The survey contained from 0.5-0.6%, the essential oil was light yellow in color and had a slight fragrance. The main chemical constituents of essential oil are Estragole (92.16%), 1,8-Cineole (1.69%), 3-Carene (0.99%) and trans-α-Bergamotene (0.52%).

Essential oil has very strong antioxidant capacity, DPPH free radical inhibition rate is 94.40% with IC50 value = 1.08 mg/ml and ability to inhibit *E. coli* is 89.20% with an antibacterial diameter of 48.2 mm. Further studies will contribute to opening up the potential of *Ocimum basilicum* L. into functional foods and pharmaceuticals, contributing to the effective exploitation and use of this plant.

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ISOLATION OF SOME COMPOUNDS FROM ETHYL ACETATE EXTRACT FROM THE STEMS OF *ROUREA MINOR*

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Abstract: *Rourea minor is a species which belongs to the family of Connaraceae. According to traditional medicine, the species Rourea minor has biological abilities such as antibiotic, antibacterial, hemostatic and wound-healing activities. In this study, from the ethyl acetate extract of Rourea minor stems collected in Ben En national park, Thanh Hoa province, three compounds were isolated including β-sitosterol, vanilic acid and sitoindoside I. The structures of these compounds have been identified by NMR, MS spectroscopic data and comparison with the reported literature.*

Keywords: *Rourea minor, isolation, β-sitosterol, vanilic acid, sitoindoside I.*

1. Introduction

Rourea minor subsp. *microphylla* Vidal. is a climbing shrub or small tree of the *Rourea* genus belonging to the Connaraceae family which includes about 100 species [1] [2]. Studies on some species of the genus Rourea have shown that extracts and compounds isolated from this genus have antibacterial, anti-inflammatory, anti-diabetic, antioxidant, hepatoprotective, and anti-fever activities [3] [4] [5]. The plant is widely distributed in China, Vietnam, Laos, India, Sri Lanka, grows and develops in sunny areas. In Vietnam, the tree grows a lot in the middle and plateau provinces. According to folk experience, this plant is used as a tonic for women after childbirth, also used to treat yellow, red urine, urinary incontinence, and boils. The plant is harvested almost all year round and is used in both fresh and dried forms [6]. Studies on the chemical composition and biological activities of *Rourea minor* plants are quite limited. Studies of Preeti Kulkarni, Anu Chaudhary and their colleagues showed that ethanol extract, methanol extract and aqueous extract of Rourea minor root have anti-diabetic and anti-obesity effects in rats [7] [8]. A recent study by Nguyen Ngoc Hieu [9] Chemical composition of *Rourea minor* tree showed that there are phenolic compounds, alkaloids, fatty acids in this plant and two new substances, lethedocin 3′-O-β-D-glucopyranoside and 3-O-(6′–O-vanilloyl)-β-D-glucopyranosyl 4-hydroxyphenethyl alcohol [9]. To further study on the chemical composition of RM, in this study, we report the isolation of three compounds obtained from the ethyl acetate extract of this plant collected in Ben En National Park, Thanh Hoa province.

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2. Materials and Methods

2.1. Plant materials

The plant samples were collected from Ben En National Park in 2018 and identified by Dr. Do Ngoc Dai, Vinh University of Economics with the scientific name as *Rourea minor* subsp. *microphylla* Vidal., belonging to the Connariaceae family. A voucher specimen (MK-301) was deposited at the Faculty of Natural Sciences, Hong Duc University.

2.2. General experimental procedures

The ^{1}H-NMR (500 MHz) and ^{13}C-NMR (125 MHz) spectra were recorded by a Bruker AM500 FT-NMR spectrometer using TMS as an internal standard. The electrospray ionization mass spectra (ESI-MS) were obtained on an Agilent 1260 series single quadrupole LC/MS system. Column chromatography (CC) was performed on silica gel (Merck, 230-400 mesh) or Sephadex LH-20. Thin layer chromatography used precoated silica gel plates (Merck 60 F_{254}). Compounds were visualized by spraying with Ce-Mo stain. Polar rotation angle was measured on a JASCO P-2000 Polarimeter (Institute of Marine Biochemistry).

2.3. Extraction and isolation

The stem of *Rourea minor* was dried in the shade, and then ground into powder. The dried stems (3.0 kg) were extracted with MeOH (10 L) at room temperature for 24 h, then the extract was removed and the medicinal residues were further extracted with MeOH, this process was repeated 5 times. The extract was combined and concentrated under reduced pressure to obtain a MeOH residue. Dissolved the MeOH residue with 1L of distilled water, and then distributed the extract with ethylacetate solvent. The extract after removing the solvent obtained 9.8 grams of EtOAc residue.

The EtOAc residue (9.8 g) was separated on a normal-phase silica gel chromatographic column, eluting the gradient with the solvent system n-hexane/EtOAc (0-100% ethylacetate). Checked by thin layer chromatography, combined tubes of the same composition and remove the solvent to obtain the corresponding 9 fractions from K1-K9. The K2 fraction (1.0 g) was crystallized the fraction to obtain compound **1** (40 mg).

The K5 fraction (0.32 g) was purified through a silica gel column with the solvent system n-hexane/EtOAc (8/2) to get 5 fractions K5.1 to K5.5. The K5.2 fraction (15.3 mg) was purified through a normal phase silica gel column, eluted with the solvent system n-hexane /EtOAc (6/4) to obtain compound **2** (2.1 mg). The K5.5 fraction (0.11 g) was purified through a normal phase silica gel column, eluted with the n-hexane/acetone solvent system (7/3) to yield compound **3** (3.2 mg).

2.4. Properties and spectral values of the isolated compounds

β-sitosterol **(1)**: White powder, ESI-MS *m/z* 415.1 $[M+H]^{+}$. ^{1}H-NMR (500MHz, $CDCl_3$), δ (ppm): 5.34 (1H, br d, *J* = 4.5 Hz), 3.52 (1H, m), 2.20-2.31 (2H, m), 1.95-2.02

(2H, m), 1.83-1.86 (3H, m), 1.47-1.70 (9H, m), 1.20-1.38 (6H, m), 0.90-1.20 (8H, m), 1.01 (3H, s), 0.92 (3H, d, J = 6.5 Hz), 0.84 (3H, t, J = 7.5Hz), 0.83 (3H, d, J = 7.0 Hz), 0.80 (3H, d, J = 7.0 Hz), 0.68 (3H, s). ^{13}C-NMR (125 MHz, $CDCl_3$), δ (ppm): 140.7, 121.7, 71.8, 56.7, 56.0, 50.1,45.8, 42.3, 42.3, 39.8, 37.2, 36.5, 36.1, 33.9, 31.9, 31.9, 31.6, 29.1, 28.2, 26.1, 24.3, 23.1, 21.1, 19.8, 19.4, 19.0, 18.7, 11.9, 11.8.

Vanilic acid (**2**): Light yellow solid. ESI-MS *m/z* 169 $[M+H]^+$. ^{1}H-NMR (500 MHz, CD_3OD) δ (ppm): 7.58 (1H, d, J = 1 Hz, H-2), 7.56 (1H, dd, J = 2.0, 8.0 Hz, H-6), 6.85 (1H, d, J = 8.5 Hz, H-5), 3.91 (3H, s, Ome). ^{13}C-NMR (125 MHz, CD_3OD) δ (ppm): 168.0 (COOH), 151.5 (C-3), 147.6 (C-4), 124.1 (C-1), 123.0 (C-6), 116.7 (C-2), 114.7 (C-5), 55.4 (OCH_3).

Sitoindoside I (**3**): White solid. ESI-MS *m/z* 815 $[M+H]^+$. ^{1}H-NMR (500 MHz, $CDCl_3$) δ (ppm): 5.37 (1H, m, H-6), 4.48 (1H, dd, J = 12.5, 4.5 Hz, H-6′a), 4.38 (1H, d, J = 7.5 Hz, H-1′), 4.28 (1H, br d, J = 12.5Hz, H-6′b), 3.4-3.6 (5H, m, H-3, 2′, 3′, 4′, 5′), 1.01 (3H, s, H-19), 0.92 (3H, d, J = 6,5Hz, H-21), 0.88 (3H, t, J = 7.0Hz, Me-16″), 0.85 (3H, t, J = 7,5Hz, Me-29), 0.84 (3H, d, J = 6.5 Hz, Me-27), 0.82 (3H, d, J = 6.5 Hz, Me-26), 0.68 (3H, s, Me-18). ^{13}C-NMR (125 MHz, $CDCl_3$) δ (ppm): 174.7 (C-1″), 140.3 (C-5), 122.2 (C-6), 101.2 (C-1′), 79.6 (C-3), 76.0 (C-3′), 74.0 (C-5′), 73.6 (C-2′), 70.1 (C-4′), 63.2 (C-6′), 56.8 (C-14), 56.1 (C-17), 50.2 (C-9), 45.9 (C-24), 42.3 (C-13), 39.8 (C-12), 38.9 (C-4), 37.3 (C-1), 36.7 (C-10), 36.2 (C-20), 34.2 (C-2″), 34.0 (C-22), 31.9 (C-7, 8, 14″), 29.2–29.7 (C-2, 4″-13″), 28.3 (C-16), 26.1 (C-23), 25.0 (C-3″), 24.3 (C-15), 23.1 (C-28), 22.7 (C-15″), 21.1 (C-11), 19.8 (C-27), 19.4 (C-19), 19.0 (C-26), 18.8 (C-21), 14.1 (C-16″), 12.0 (C-29), 11.9 (C-18).

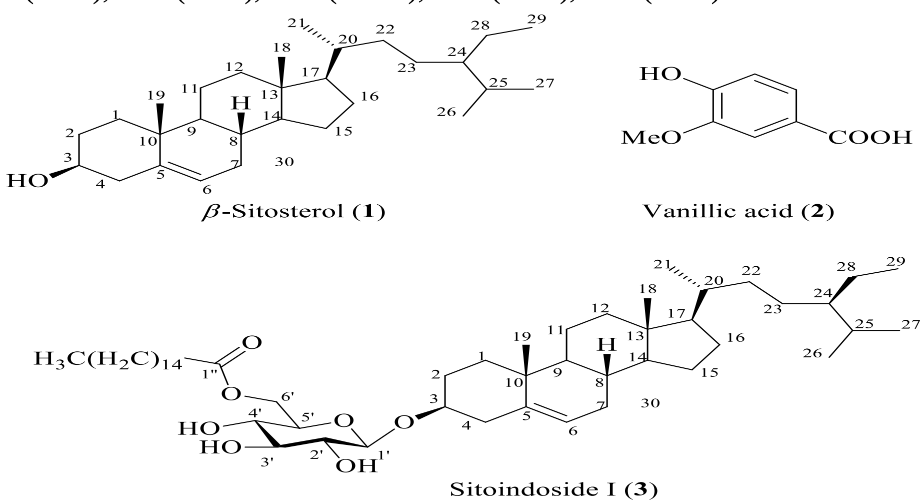

Figure 1*. Chemical structures of the isolated compounds* ***1-3***

3. Results and Discussion

3.1. Determination of the structure of β-sitosterol (1)

Compound **1** was obtained as a white powder, mass spectrometry (ESI-MS) has a pseudo molecular ion peak of *m/z* $[M+H]^+$ of 415.1, indicating that the molar mass of compound **1** is 414.1 corresponding to the molecular formula $C_{29}H_{50}O$. On the ^{1}H-NMR spectrum of **1**, there are typical signals for a sterol compound with a double bond with a signal proton at position δ_H 5.34 (br d, *J* = 5 Hz). There is a hydroxyl group with signal CH-OH at position δ_H 3.52 (1H, m) and 6 methyl groups including 2 singlets at δ_H 0.68 (3H, s) and 1.01 (3H, s), 3 doublet signals at δ 0.92 (3H, d, *J*= 6.5 Hz), 0.83 (3H, d, *J*= 7.3 Hz), 0.80 (3H, d, *J*= 6.8 Hz), and one triplet signal at δ 0.84 (3H, t, *J*= 7.5 Hz). In addition, other signals of the sterol ring, ^{13}C-NMR and DEPT spectra for **1** have 29 carbons, including 6 CH_3 groups, 11 CH_2 groups, 9 CH groups and 3 quaternary carbons. From the physical properties, the ^{1}H-NMR, ^{13}C-NMR, ESI-MS spectral data of **1** and the comparison of the spectral data with reference material is suggested that this compound is *β*-sitosterol [10].

3.2. Determination of the structure of vanillic acid (2)

Compound **2** was isolated as a pale yellow solid. ESI-MS mass spectroscopy for pseudo-molecular ion peak *m/z* 169 $[M+H]^+$ allows prediction of compound 3 with molecular formula $C_8H_8O_4$ (M = 168). On the ^{1}H-NMR spectrum of compound **2**, the resonance signal of 3 aromatic ring protons of the ABX system appears at δ_H 7.58 (1H, d, J = 1 Hz, H-2), 7.56 (1H, dd, *J* = 2.0) , 8.0 Hz, H-6), 6.85 (1H, d, *J* = 8.5 Hz, H-5) and 1 methoxy group. ^{13}C-NMR spectrum shows carboxylic acid group signal at 168.0 (COOH) and 6 aromatic carbon signal at 151.5 (C-3), 147.6 (C-4), 124.1 (C-1), 123.0 (C-6), 116.7 (C-2), 114.7 (C-5), methoxy group at 55.4 (OCH_3). From the above analyzed mass spectrometry and nuclear magnetic resonance spectroscopy data and compared with reference literature [9], compound 2 was identified as a vanillic acid or 4-hydroxy-3-methoxybenzoic acid compound.

3.3. Determination of the structure of sitoindosid I (3)

Compound **3** is a white solid. ESI-MS spectrum gives pseudo molecular ion peak at *m/z* $[M+H]^+$ 815, corresponding to the molecular formula $C_{51}H_{90}O_7$ (M=814). On the ^{1}H-NMR spectrum, there are signals specific to sterol glucoside ester compounds. The sterol framework signals were observed with a double bond at δ_H 5.37 (1H, m, H-6); an oxymethin group at δ_H 3.5 (1H, m, H-3) and 6 methyl groups consisting of 2 singlets at δ_H 0.68 (3H, s) and 1.01 (3H, s), 3 doublets at δ_H 0, 92 (3H, d, J = 6.5 Hz), 0.84 (3H, d, J = 6.5Hz), 0.82 (3H, d, *J* = 6.5 Hz), and a triplet at δ_H 0.85 (3H, t, *J* = 7.5Hz). The signal of glucose molecule was detected by 4 oxymethin groups at position δ_H 3.4-3.6 (4H, m), oxymethylene group signal at δ_H 4.48 (1H, dd, *J* = 12.5, 4.5 Hz, H-6′a) and 4.28 (1H, br d, *J* = 12.5 Hz, H-6′b); and the sugar anomer proton signal at δ_H 4.38 (1H, d, *J* = 7.5 Hz). The signal of a fatty acid was detected based on the CH_2 group signal at δ_H 2.35 (2H, t, *J*=7.5 Hz), the long-terminal CH_3 methyl group at δ_H 0.88 position (3H, t). , J = 7.0 Hz) and long-chain CH_2 groups signal at δ_H 1.25 (m). On the ^{13}C-NMR and DEPT spectra, there are also signals characteristic of sterol glucoside ester compound. Signals of the sterol nucleus include 6 methyl groups at δ_C 19.8 (C-27), 19.4 (C-19), 19.0 (C-26), 18.8 (C-21), 12, 0 (C-29), 11.9 (C-18); oxymethin group at δC 79.6 (C-3) and signal of 2 olefinic carbon at δ_C 140.3 (C-5) and 122.2 (C-6). The signal of the sugar group consists of a carbon anomer signal at δ_C 101.2 (C-1′) along with 5 other signals of 1 oxymethylene group and 4 oxymethin groups at δ_C 76.0 (C-3′), 74.0 (C-5′), 73.6 (C-2′), 70.1 (C-4′) and 63.2 (C-6′). The ester functional group was detected by the C=O group signal at δC 174.7 (C-1″). Other long-chain fatty acid signals include a signal at δ_C 34.2 (C-2″), a saturated circuit signal at CH_2 29.2-29.7 (C-4″-13″) and a methyl group, at δ_C 14.1 (C-16″). From the data of mass spectrometry and NMR spectrum analyzed above, combined with comparison with published material [12], compound **3** was identified as sitoindoside I.

4. Conclusion

A phytochemical investigation of the ethyl acetate extract of the stems of *Rourea minor* led to the isolation of three compounds including β-sitosterol, vanilic acid and sitoindoside I. This is the first study on the chemical composition of *Rourea minor* species collected in Ben En National Park, Thanh Hoa province. Investigation on the chemical composition as well as biological activities of this species will be continued in the near future.

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FACTORS AFFECTING CUSTOMER SATISFACTION OF FIBER - TO -THE - HOME SERVICES OF VIETTEL IN THANH HOA CITY

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Abstract: *The purpose of this paper is to investigate the factors that can affect customer satisfaction of Fiber-To-The-home (FTTH) Services of Viettel in Thanh Hoa city. Data from the study were collected from 360 customers using the Viettel FTTH Services. The study results showed that the factors affecting the FTTH Services of Viettel in Thanh Hoa city including: Reliability, Tangibles, Convenience, Service prices, Promotion policies and the Staffs. In which, Reliability is the factor that has the strongest influence on customer satisfaction. The second strongest factor influencing customer satisfaction is Tangible factor, the following factors are: Promotion policies, Service prices, Staffs and Convenience. All factors have a positive impact on customer satisfaction. The authors proposed some solutions to improve customer satisfaction about FTTH service of Viettel in Thanh Hoa city in the next time.*

Keywords: *Customer Satisfaction, Factors Influencing Customer Satisfaction, FTTH Services, Viettel, Thanh Hoa.*

1. Introduction

Customer satisfaction can be defined as a feature or characteristic that can fulfill the either a need or want of a consumer in a better way than competitors. If a company provides a product according to the requirements of their consumers, it will lead the satisfaction of them. The higher or lower satisfaction of a consumer will depend upon the quality of brand characteristics offered by a company [6]. In the study of Bodet, G. (2008), customer satisfaction is the key factor which is used to measure the company internal and external performances and assigning funds to each and every activity. The author stated that customer satisfaction leads customer loyalt, and described in their study that customer satisfaction is an important indicator for the customer loyalty. According to Shankar, V. et al (2003), the comprehend contradiction among former expectations and the actual performance of the product can be defined as customer satisfaction. In marketing, the marrow thought is customer satisfaction because it indicates the customer loyalty towards any service or product [9], they said that "customer satisfaction is worthless. Customer loyalty is priceless". The basic component of business success is customer satisfaction.

FTTH is a broadband telecommunications service by means of fiber optic cable that is connected to the door to provide high speed services such as telephone, high speed

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2 Viettel Group, Thanh Hoa Brand

Internet and television. By deploying fiber optic cable to customers' homes, the network speed will thus increase many times. The fiber optic network is brought to the subscriber's address to help customers use multiple services on the high-quality telecommunications network, including entertainment television services. The transmission line has a stable speed; high speed Internet access, no signal loss due to electromagnetic interference, weather or cable length. With FTTH technology, service providers can deliver download speeds of up to 10 Gigabits per second that are 200 times faster than ADSL [17].

In the era of competition and integration, the growth rate of FTTH services still has a lot of potential, Thanh Hoa City is a rapidly growing market, so the competition between service providers is very fierce because there are many different service providers such as Viettel, Vinaphone, FPT, etc. The level of competition is clearly shown through the continuous reduction of freight rates, the continuous launch of promotions for customers, and increase market share.

In order to contribute to improving competitiveness, maintaining the market share and further development, the authors have thoughts on how to increase customer satisfaction in order to retain long-term customers using Viettel's FTTH services and reduce the number of customers leaving the network. Therefore, this research aim to determine which factors affecting the customer satisfaction using Viettel FTTH services as well as the level of impact of each factor on customer satisfaction in order to propose some solutions to improve customer satisfaction about FTTH service of Viettel in Thanh Hoa city in the next time.

2. Literature review and proposed research model

2.1. Research models of service quality and customer satisfaction

The SERVQUAL model of Parasuraman, Zeithaml, and Berry (1988)

The SERVQUAL model, created by Parasuraman et al. in 1988, was used to evaluate customer perceptions of service quality including five dimensions (reliability, responsiveness, assurance, empathy, tangibles). These dimensions are illustrated in Table 1.

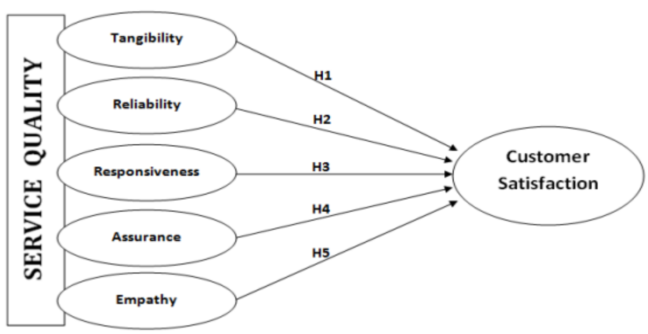
Table 1. *Dimensions of SERVQUAL model*

No.	Dimensions	Definition
1	Reliability	Express the ability to deliver timely and consistent services
2	Responsiveness	Express through eagerness to help customers and quickly resolve problems when errors or unexpected situations occur.
3	Assurance	Express through ability to build customer trust through professionalism, politeness, respect for customers, communication skills and attentive attitude to do the best support for our customers.
4	Empathy	Express through the style of services of employees through attention, care, ability to understand the needs and create feelings of secure and safety for customers.
5	Tangibles	Express through the appearance of facilities, equipment, staff uniforms, items and materials used for performing the services.

Source: Parasuraman et al. (1988)

This approach is inconsistent with some research results, which show that customer's expectations should be based on experiences. What's more, higher level of perceived service quality only sometimes is a cause of increased consumer satisfaction. In many cases, the satisfaction is an antecedent of service quality.

SERVPERF Model: Cronin and Taylor (1992) in their empirical work controverted the framework of Parasuraman, Zeithaml and Berry (1985, 1988) with respect to conceptualization and measurement of service quality, and propounded a performance-based measure of service quality called 'SERVPERF' illustrating that service quality is a form of consumer attitude. They argued that SERVPERF was an enhanced means of measuring the service quality construct. Their study was later replicated and findings suggest that little if any theoretical or empirical evidence supports the relevance of the E-P = quality gap as the basis for measuring service quality. The SERVPERF measures quality as an attitude, not satisfaction. However it uses an idea of perceived service quality leading to satisfaction. But it goes further, and connects satisfaction with further purchase intentions. The SERVPERF is a modification of SERVQUAL, and thus uses the same categories to assess service quality:



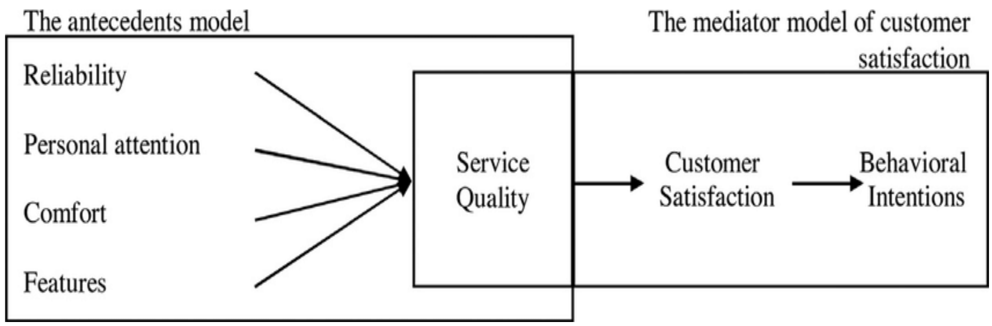
Source: Cronin and Taylor, 1992.

***Figure 1**. SERVPERF Model*

In SERVQUAL model, service quality is linked to the concepts of disconfirmation or gap between customers' perceptions and expectations. Even though it is intuitively appealing and conceptually sensible, the ability of these scores to provide additional information beyond that already contained in the perception component is under doubt. While the perception is definable and measurable in a straightforward manner as the customers' belief about service is experienced, expectation is subject to multiple interpretations and as such has been operationalized differently by different researchers. It is argued that the conceptual basis of the SERVQUAL scale is confusing with the service satisfaction and suggested to leave the perception alone; hence the SERVPERF model plays its role.

The antecedents and mediator model of Dabholkar, Shepherd, and Thorpe (2000)

This model can be considered as a comprehensive model of quality of service, including the consideration of the antecedents, mediators and result to provide a deeper understanding of interrelated concepts of quality of service. This model examines the concepts of service quality with the four factors: Reliability, Personal attention, Comfort, Feature that affecting service quality as a component of antecedents and the relationship between service quality with customer satisfaction and behavioral intentions. This model is illustrated in the following Fig.2.



Source: Dabholkar et al., 2000.

Figure 2. *The antecedents and mediator model*

This study finds that factors relevant to service quality are better conceived as its antecedents rather than its components and that customer satisfaction strongly mediates the effect of service quality on behavioral intentions.

2.2. Proposed research model

On the basis of SERVPERF model and previous similar research results, the authors adjust the content of six basic components of service quality to suit the telecommunications service industry. The model has the following six factors: (1) Reliability, (2) Tangibles, (3) Convenience, (4) Service prices, (5) Promotion policies, and (6) Staffs.

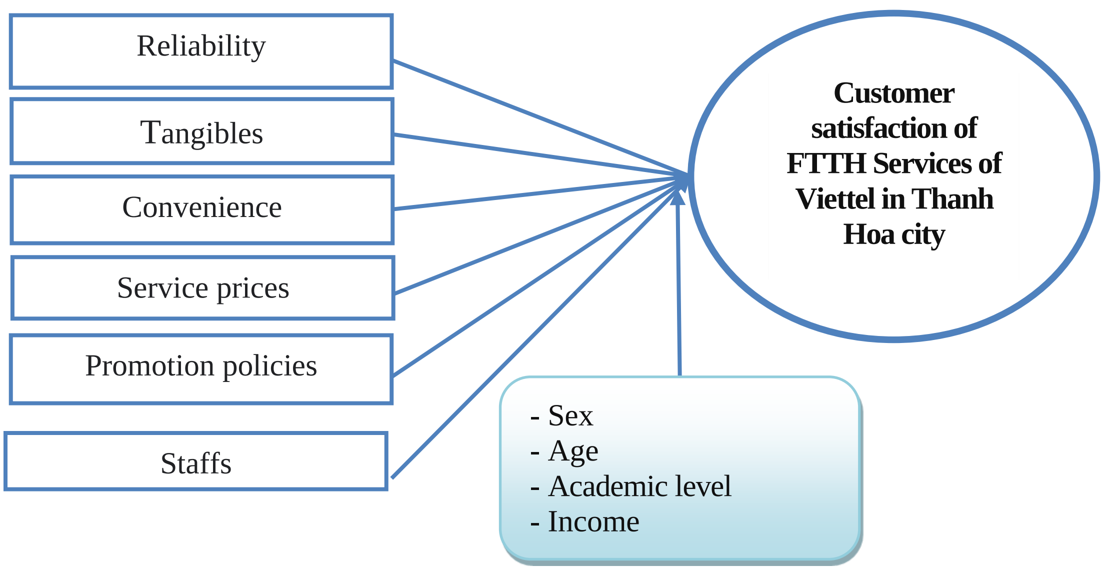


Figure 3. *Proposed research model*

Based on the proposed research model, the hypothesis of the Relationship between customer satisfaction and its determinants are as follows:

H1: Reliability has a positive impact on customer satisfaction

H2: Tangibles positively affect customer satisfaction

H3: Convenience has a positive impact on customer satisfaction

H4: Service prices positively affects customer satisfaction

H5: Promotion policies have a positive effect on customer satisfaction

H6: Staffs have a positive impact on customer satisfaction

3. Methodology

3.1. Research Method

We use both qualitative and quantitative approaches through focus group discussion and direct interviews with 360 customers using FTTH Services of Viettel in Thanh Hoa city. The multiregression analysis method is employed to analyze the data obtained from surveys. The data collected were reviewed during and after collection, uncoded variables were coded, classified, and tabulated. Statistical Package (SPSS) version 22.0 was used for analysis of data.

Qualitative research method to determine the factors affecting customer satisfaction with Viettel's FTTH service quality in Thanh Hoa city. With the limit time and resource to complete the study, this research only used the in-depth focus group discussion techniques combining with the synthesis of the author about the related theories, studies, the model and scales that are applied in measuring the quality of service, customer satisfaction. In the first step, the author synthesized and adopted the scale from previous studies,translated and then modified the wordings to suitable with the objects for survey to make a draft questionnaire. After that, the author gathered a group of nine people who are 01 deputy director, 03 department heads, 03 deputy heads, 02 specialists invited to the discussion to adjust the observation variables. All the ideas and suggested modifications in the discussion are used to adjust the questionnaire and form the official scale. This preliminary study was conducted in Thanh Hoa City from October to December 2021.

Table 2. *Scale Development*

No.	Code	Statements	Sourse
	Independent Variables		
	Rel	***Reliability***	Moon-Koo Kim et. al. (2004)
1	Rel1	The speed of connection to the network is in accordance with the commitment to the customers	
2	Rel2	Payment invoices are always clear and accurate	
3	Rel3	Information for customers is accurate and in time	
4	Rel4	Providing the service as commitment to the customers	
5	Rel5	Time to repair transmission line is as commitment	
	Tan	***Tangible factors***	Cronin and Taylor (1992)
6	Tan1	Viettel has a system of transaction points located in convenient locations for customers to easily find	

7	Tan 2	The transaction points are spacious, reasonably arranged, creating trust for customers.	Parasuraman et. al. (1988)
8	Tan 3	Euipments at transaction points are modern and easy-to-use	Cronin and Taylor (1992)
9	Tan 4	Professionally designed website, easy to use	
	Pri	***Price service***	
10	Pri 1	The price of FTTH connection and installation is reasonable	Built by The Authors
11	Pri 2	The price is suitable for the customer's pocket	Moon-Koo Kim et. al. (2004)
12	Pri 3	There are many packages with suitable prices for customers	
13	Pri 4	The price is suitable with the transmission speed.	
	Pro	***Promotion policies***	Elizabeth Levin (2018)
14	Pro1	The Company uses television, radio, newspapers, and social networks to announce promotions and discounts regularly	
15	Pro 2	Many gifts, attractive promotions included	
16	Pro 3	Promotion period lasts long time	
17	Pro 4	The content of the promotions is really attractive	
	Con	***Convenience***	Moon-Koo Kim et. al. (2004) Elizabeth Levin (2018)
18	Con 1	Customers can easily call the customer care center for service support	
19	Con 1	The registration and using process FTTH service is very easy	
20	Con 1	Procedures for service restoration (cancellation, re-registration of packages), and service changes are easy for users	
21	Con 1	Easy to change package of FTTH service when the customer demand changes	
	Sta	***Staffs***	Dabholkar et. al. (2000) Moon-Koo Kim et al. (2004)
22	Sta1	Staffs give clear advice on the prices, technical solutions to help customers choose the most suitable solution according to their needs.	
23	Sta2	Staffs install and repair the FTTH service staff professionally	
24	Sta3	Staffs always listen and understand the needs and desires of customers.	
25	Sta4	Staffs maintain close relationships with customers	
	Dependent Variables		Moon-Koo Kim et. al. (2004)
	CS	Customer satisfaction	
26	CS1	I am satisfied with the quality of service using	
27	CS2	I will continue to use this service in the future	
28	CS3	I will recommend this service to friends and family	

Source: The synthesis of the authors

It can be seen that the proposed model to assess customer satisfaction for FTTH services has 6 groups of independent factors with 25 measurement variables and 01 dependent factor with 3 measurement variables.

Quantitative research method is a method used to determine the influence of the main factors affecting customer satisfaction. This quantitative research was conducted through direct interviews with 360 customers who are using FTTH services of Viettel in Thanh Hoa city by a detailed questionnaire. The collection of information is directly done by sending questionnaires to customers who are using Viettel's FTTH service in Thanh Hoa city. The main quantitative methods used are Cronbach Alpha reliability test, EFA exploratory factor analysis, correlation and Multi-regression analysis through SPSS.22 software. The purpose of regression analysis is to predict the effect level of independent variables to the dependent variable [8].

3.2. Sampling method

The sample was chosen according to the convenience sampling method (non-probability). After carefully considering about time, budget and resource available of this research, the author chose non-probability method with form of convenience sampling.

3.3. Sample size

According to Hair et al. (1998), for exploratory factor analysis (EFA), the minimum sample size is $N \geq 5*x$ (x: total number of observed variables). In multiple linear regression analysis, according to Tabachnick and Fidell (1991), the sample size must satisfy the formula $N \geq 8*x + 50$. In this study, the authors choose a large enough sample size to satisfy both conditions. So the minimum sample size is N ≥ max (sample size required by EFA; sample size required by multiple regression). The survey questionnaire in the official study included 25 observational variables for service quality and its components with 3 observational variables for customer satisfaction. Therefore, the minimum sample size is N = max (5*28; 8*28 + 50) = 274 questionnaires. However, to ensure the reliability of the results as required, the authors sent out 360 survey questionnaires. Total obtained: 325 valid questionnaires were processed.

4. Results and Discussion

4.1. Reliability of Scale

All observed variables of this scale have total correlation coefficients greater than 0.3, so they are all used for subsequent EFA analysis. The results after analyzing Cronbach's Alpha coefficient and removing observed variables that do not guarantee reliability, the scale of factors affecting customer satisfaction is measured by 22 observed variables for 6 components. (compared to the initial 25 observed variables for 6 factors affecting satisfaction) and the satisfaction scale is measured by 3 observed variables (remaining the same compared to the original number of observed variables). Therefore, all the scales of consumer satisfaction after checking the reliability of Cronbach's Alpha will be included in the EFA factor analysis. Through evaluating the reliability of the scales, the authors proceed to remove the inappropriate variables as 03 variables as Rel1, Sta3, Tan4, the remaining variables are summarized in the following table:

Table 3. *Reliability of the Scale of the study*

Code	Factors	Before Cronbach's Alpha Test	After Cronbach's Alpha Test	
		N. of Items	***Cronbach's Alpha***	***N. of Items left if Item Deleted***
Con	Convenience	4	0.839	4
Pro	Promotion policies	4	0.880	4
Rel	Reliability	5	0.906	4 (delete Rel1)
Sta	Staffs	4	0.720	3 (delete Staf3)
Pri	Service prices	4	0.873	4
Tan	Tangibles	4	0.745	3 (delete Tan 4)
CS	Customer satisfaction	3	0.842	3

4.2. Exploratory Factor Analysis (EFA) results

KMO analysis and Barrlet's test of the independent variables

Through the analysis results of Table 4, KMO reached 0.868. Thus, the KMO index of the research model is greater than 0.5, showing that the application of exploratory factor analysis here is completely appropriate.

Table 4. *KMO and Bartlett's Test*

Kaiser-Meyer-Olkin	Measure of Sampling Adequacy.	.868
Bartlett's Test of Sphericity	Approx. Chi-Square	3819.553
	Df	231
	Sig.	.000

Based on the table of results, we can see that the sample data is completely consistent with the method of exploratory factor analysis (EFA).

Exploratory Factor Analysis (EFA) results for customer satisfaction

The customer satisfaction scale is grouped in one measured factors and EFA results are used by Varimax rotation method. The Exploratory Factor Analysis (EFA) results are shown in Table 5. The Bartlett test results in the KMO and Bartlett's test with Sig = 0.000 and the KMO = 0.727 > 0.5 show that factor analysis is appropriate, meeting the requirements. EFA of customer satisfaction is grouped into one factor with Principal components extraction.

Table 5. *EFA results for CS*

Scales	Factor loading	Test Types	Value
CS1	.878	KMO	0.727
CS2	.876	Sig.	0.000
CS3	.860	Quoted variance	75.929%

4.*3. Pearson correlation analysis*

After analyzing the collected data through the steps of Cronbach' alpha reliability test and Exploratory Factor Analysis, the authors analysis the results with Pearson correlation analysis by SPSS.22 as follows.

Table 6. *Pearson correlation analysis*

		CS	Con	Pro	Rel	Sta	Pri	Tan
CS	Pearson Correlation	1	.731**	.525**	.606**	.612**	.408**	.507**
	Sig. (2-tailed)		.000	.000	.000	.000	.000	.000
	N	325	325	325	325	325	325	325
Con	Pearson Correlation	.731**	1	.345**	.549**	.519**	.311**	.319**
	Sig. (2-tailed)	.000		.000	.000	.000	.000	.000
	N	325	325	325	325	325	325	325
Pro	Pearson Correlation	.525**	.345**	1	.385**	.352**	.216**	.446**
	Sig. (2-tailed)	.000	.000		.000	.000	.000	.000
	N	325	325	325	325	325	325	325
Rel	Pearson Correlation	.606**	.549**	.385**	1	.473**	.259**	.369**
	Sig. (2-tailed)	.000	.000	.000		.000	.000	.000
	N	325	325	325	325	325	325	325
Sta	Pearson Correlation	.612**	.519**	.352**	.473**	1	.323**	.451**
	Sig. (2-tailed)	.000	.000	.000	.000		.000	.000
	N	325	325	325	325	325	325	325
Pri	Pearson Correlation	.408**	.311**	.216**	.259**	.323**	1	.210**
	Sig. (2-tailed)	.000	.000	.000	.000	.000		.000
	N	325	325	325	325	325	325	325
Tan	Pearson Correlation	.507**	.319**	.446**	.369**	.451**	.210**	1
	Sig. (2-tailed)	.000	.000	.000	.000	.000	.000	
	N	325	325	325	325	325	325	325

The results show that the Sig. values between the dependent variable (CS) and 06 independent variables are all equal 0.000 < 0.05. Therefore, the variables are all correlated with the dependent variable and are statistically significant. The strongest correlation with the dependent variable is Con - Conveniece (with the correlation coefficient is 0.731, with p <0.05) and the weakest correlation with the dependent variable is Pri-Service prices, the correlation efficient is: 0.408, with p<0.05). Among the independent variables, the correlation coefficients are all less than 0.3, so the possibility of a linear relationship between the independent variables in the model is unlikely.

4.4. Regression analysis

Determining which factors affecting customer satisfaction of Viettel's FTTH service in Thanh Hoa city is one of the important issues to help propose solutions to improve customer satisfaction. Because through this process, there is a basis to come up with solutions to further improve customer satisfaction about the quality of Viettel's FTTH service, thereby contributing to improving the efficiency.

Table 7. *Model Summary*

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Durbin-Watson
1	.842[a]	.709	.704	.428	1.846

The results of linear regression analysis show that the model has R square equal 0.709 and adjusted R square equal 0.704. This result shows that the appropriateness of the model is 70.4%, in other words, 70.4% of the variation of Customer Satisfaction is explained by 6 factors: Convenience, Promotion policies, Reliability, Staffs, Service Prices and Tangibles. On the other hand, Durbin-Watson statistic equal 1.846 (>1) indicates that there is no autocorrelation in the model.

Table 8. *Anova Analysis*

Model		Sum of Squares	Df	Mean Square	F	Sig.
1	Regression	142.343	6	23.724	129.393	.000[b]
	Residual	58.304	318	.183		
	Total	200.648	324			

The results from table 8 show that the value of F - statistic equal 129.393, with the Sig. value less than 0.5 which means that the regression model of Customer Satisfaction is suitable, at five percent significance level the combination of factors included in the model can explain the change of the dependent variable.

Besides, the results of the regression analysis indicate that all the independent variables have Sig. less than 0.05. that is, all independent factors affect customer satisfaction about FTTH service of Viettel in Thanh Hoa city. The importance of each factor depends on the standardized Beta coefficient (in terms of the absolute value of the coefficient), or which factor has a higher standardized Beta coefficient, which has a stronger impact on customer satisfaction.

Table 9. *Regression Results of the Independent and Dependent Variables*

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Collinearity Statistics	
		B	Std. Error	Beta			Tolerance	VIF
1	(Constant)	-.663	.169		-3.926	.000		
	Rel	.431	.040	.425	10.840	.000	.594	1.683
	Con	.124	.034	.119	3.642	.000	.859	1.164
	Tan	.189	.038	.178	5.015	.000	.727	1.376
	Pri	.146	.038	.146	3.798	.000	.615	1.627
	Pro	.159	.040	.157	4.021	.000	.599	1.670
	Sta	.145	.037	.142	3.916	.000	.692	1.445

The regression coefficients show that the Sig. values of the independent factors are less than 5%, this proves that all 6 factors: Rel, Con, Pro, Sta, Pri, Tan are all staticsally significant in the model and all have an impact on Customer Satisfaction. Besides, the VIF magnification factor is in the permissible condition (If the VIF is greater than 10, there is a sign of multicollinearity), this proves that the multicollinearity between the independent variables is very low, consistent with the research assumption.

We have the following regression equation:

CS = - 0.663 +0.425 Rel + 0.119 Con+ 0.178 Tan + 0.146 Pri+ 0.157 Pro+0.142 Sta

In which: CS: Customer satisfaction; Con: Convenience; Pro: Promotion policies; Rel: Reliability; Sta: Staffs; Pri: Service prices; Tan: Tangibles.

The effect level of independent factors on customer satisfaction is determined through the standardized beta coefficient, as follows: Reliability is the factor that has the strongest influence on customer satisfaction (standardized β coefficient = 0.425). Similarly, the second strongest factor affecting consumer satisfaction is Tangibles factors (standardized β coefficient = 0.178), the following factors are Promotion policies (standardized β coefficient = 0.157); Service Prices (normalized β coefficient = 0.146); Staffs (standardized β coefficient = 0.142) and Convenience (standardized β coefficient = 0.119). All of independent factors have a positive impact on customer satisfaction. Therefore, the hypotheses H1, H2, H3, H4, H5, H6 of the research are accepted.

Table 10. *Summary of the results of testing the research hypotheses*

Hypothesis	Predictors	Dependent Variable	Statistical relationship	Coefficients	No. of effect	Result
H1	Reliability	Customer satisfaction	+	0.425	1	Accepted
H2	Tangible factors		+	0.178	2	Accepted
H3	Price services		+	0.146	4	Accepted
H4	Promotion polycies		+	0.157	3	Accepted
H5	Convenience		+	0.119	6	Accepted
H6	Staffs		+	0.142	5	Accepted

5. Conclusions and Recommendations

Through the research, six important factors affecting customer satisfaction about FTTH services of Viettel in Thanh Hoa city have been identified, including: Reliability, Tangibles, Service prices, Promotion policies, Staffs and Convenience. Understanding these factors will help the company have a more holistic view of psychology, needs and understand the feelings of customers when using FTTH service so that the company can build a successful marketing strategy to improve the market share in the future.

Based on the results, the authors propose some recommendations to improve customer satisfaction using FTTH service of Viettel in Thanh Hoa city in the near future, as follows:

5.1. Regarding the Reliability Solution

Reliability is an extremely important factor effecting customer satisfaction, therefore, the company should concentrate on some solutions which improve the company's reliability.

Good customer care: Caring for customers using FTTH service, the customer service department regularly calls a random number of customers in each district that needs to be surveyed to collect customer reviews about the service. Every week, there will be a statement on these comments to have a plan to improve service quality. Capture and in-depth understanding of competitors' services, thereby creating different, better and more affordable services.

Speeding up the process of putting into operation the newly built undersea fiber optic cable system in order to minimize unfortunate incidents caused by cable breaks. Enhancing the network user experience.

Establishing a quick troubleshooting team specialized in handling problems related to FTTH service as well as customer problems. Arranging staffs to handle incidents with high expertise, professional operation, quick response and flexible work handling.

5.2. Regarding the Tangibles Solution

According to the research results, the factor of tangibles is the second factor affecting customer satisfaction. Therefore, Viettel Thanh Hoa needs to take reasonable measures to improve and complete the tangible means, thereby increasing the quality of FTTH services to customers.

For agents, it is necessary to fully equip the system of signs, logos, price lists, means of communication, leaflets, advertising banners, book systems, as well as support parts of the funding, help agents redecorate the transaction points, and purchase necessary equipments to ensure that the agent's transaction points meet the standards of Viettel Thanh Hoa. The specific level of support will depend on the location and potential of the agent.

However, in order to ensure investment efficiency and closely associate the agent with the unit, it is also necessary to have clear regulations on the time and effectiveness of the agent's operation, in case of a breach of the contract the amount this investment will be deducted from the deposit.

For retail stores: Equipped with zonal banners, banners, books to record customer information and track sales, means of communication.

For toll collection collaborators: Equip tools such as bill bags, books to record customer information and monitor the collection of freight.

5.3. Regarding the Promotion Policy solutions

Offering some policies for loyal customers such as giving souvenirs if that customer has used the service for more than 5 years, for example.

Giving gifts to loyal customers on holidays and birthdays in some simple forms such as sending greeting cards or giving a part of the package in that month to customers but giving customers the feeling of being care.

Strengthening advertising and marketing for competitive services and new services on highly effective means such as: facebook, youtube, marketing channels on search engines, TV channels of the city.

Maintaining practical promotions for customers such as: free initial installation.

5.4. Regarding the Staff Solutions

The staff is the most important factor to be able to develop the business and is the group of factors that bring the highest satisfaction to customers. Through the analysis results, it shows that customers are really satisfied with the staffs of Viettel Thanh Hoa. Consultants, transaction staffs, and call center staffs are the whole face of the business because they are always the ones in contact with customers.

Improving the work of supplementing human resources, strengthening the training and inspection of customer care staff, training and ensuring staff have professional knowledge.

Training on communication skills, sales skills, management skills, the application of online tools to business activities for customer care staffs through short term training courses.

Paying attention to recruitment, Viettel Thanh Hoa must choose people who have the ability to negotiate with customers. Paying special attention and choose those who understand the customer's psychology well, have ethics and standard manners.

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ON THE C_g-ASYMPTOTIC EQUIVALENCE OF DIFFERENTIAL EQUATIONS WITH MAXIMA

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Abstract: *The aim of this paper is to investigate the* C_g*-asymptotic equivalence of differential equations with maximum by applying the Banach's fixed point theorem in an appropriately weighted function space. In some contexts, this paper generalizes the results of D. Otrocol [8].*

Keywords: *Differential equations with maximum, asymptotic equivalence.*

1. Introduction

Differential equations with "maxima" are a special type of differential equations that contain the maximum of the unknown function over a previous interval(s). Such equations adequately model real world processes whose present state significantly depends on the maximum value of the state on a past time interval. For example, in the theory of automatic control of various technical systems, it often occurs that the law of regulation depends on the maximum values of some regulated state parameters over certain time intervals. In [6], E. P. Popov considered the system for regulating the voltage of a generator of constant current. The object of the experiment was a generator of constant current with parallel simulation and the regulated quantity was the voltage at the source electric current. The equation describing the work of the regulator involves the maximum of the unknown function and it has the form

$$T_0 u'(t) = -u(t) - q \max_{s\in[t-h,t]} u(s) + f(t), \tag{1.1}$$

where T_0 and q are constants characterizing the object, $u(t)$ is the regulated voltage and $f(t)$ is the effect of the perturbation that appears associated to the change of voltage.

Recently, the interest in differential equations with "maxima" has increased exponentially. Let us mention, for instance, iteration method for approximating solutions of nonlinear differential equations with maxima [1] [2] [3]; qualitative properties of solutions [4] [5] [9] [10], etc.

Let $\mathbb{R}^n$ be the Euclidian n-space. For $u=(u_1,u_2,\cdots,u_n)^T \in \mathbb{R}^n$, let $\| u \| := \max\{|u_1|,\cdots,|u_n|\}$ be the norm of u. For a matrix $A \in M_{n\times n}(\mathbb{R}), A=(a_{ij})$, we define the norm $|A|$ of A by $|A| = \max_{1\leqslant i\leqslant n} \sum_{j=1}^{n} |a_{ij}|$.

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In this paper, we consider the following differential system

$$\dot{x}(t) = A(t)x(t), \quad t \geqslant t_0 \tag{1.2}$$

and the perturbed one with maxima of the form

$$\dot{y}(t) = A(t)y(t) + f\left(t, y(t), \max_{\xi \in [t_0, t]} y(\xi)\right), \quad t \geqslant t_0. \tag{1.3}$$

More precisely, the following Cauchy problem

$$y'(t) = A(t)y(t) + f\left(t, y(t), \max_{t_0 \le \xi \le t} y(\xi)\right), \ t \in [t_0, \infty) \tag{1.4}$$

$$y(t_0) = y_0. \tag{1.5}$$

Definition 1.1 ([8]). The Equations (1.2) and (1.3) are asymptotically equivalent if for every solution x of (1.2), there is a solution y of (1.3) such that

$$\lim_{t \to \infty} \| x(t) - y(t) \| = 0, \tag{1.6}$$

and conversely, for each solution y of (1.3) there exists a solution x of (1.2) such that (1.6) holds. In [8], Otrocol proved the asymptotical equivalence of (1.2) and (1.3). Impressed by more general concept of this notion which introduced in [7] by Olaru called " C_g - asymptotic equivalence", in this paper we extend the result of Otrocol to the case of C_g - asymptotic equivalence. To do this, we make the following assumption.

Assumption 1.2. There exists $L_f : [a, \infty) \to \mathbb{R}_+$ such that

$$\| f(t, u_1, u_2) - f(t, v_1, v_2) \| \le L_f(t) \max\{| u_1 - v_1 |, | u_2 - v_2 |\}, \ \forall t \in [t_0, \infty), \ u_i, v_i \in \mathbb{R}^n, i = 1,2;$$

2. The existence of solution in weighted space

Now, for g is a given continuous and positive function defined on $[t_0, \infty)$, we define

$$BC_g = \left\{ u \in C\left([t_0, \infty), \mathbb{R}^n\right) : \sup_{t \geqslant t_0} g^{-1}(t) \| u(t) \| < \infty \right\}.$$

the Banach space endowed with the norm

$$\| x \|_{BC_g} = \sup_{t \geqslant t_0} | g^{-1}(t) x(t) |, \ \forall x \in BC_g.$$

Theorem 2.1. Let $X(t)$ be a fundamental matrix of the system (1.2). Suppose that

a) the Assumption 1.2 holds;

b) there exists a constant $K > 0$ such that

$$\| g^{-1}(t) X(t) X^{-1}(s) g(s) \| \le K, \ t_0 \le s \le t < \infty;$$

$c) \int_{t_0}^{\infty} \| g^{-1}(s) f(s, 0, 0) \| ds < \infty$;

$d) K \int_{t_0}^{\infty} L_f(s) ds < 1$.

Then the problem (1.4) - (1.5) has a unique solution in $BC_g([t_0,\infty),\mathbb{R}^n)$.

Proof. For $y\in BC_g([t_0,\infty),\mathbb{R}^n)$ we put

$$(Fy)(t):=X(t)X^{-1}(t_0)y_0+\int_{t_0}^{t}X(t)X^{-1}(s)f\left(s,y(s),\max_{t_0\leqslant\xi\leqslant s}y(\xi)\right)ds,\ \ \forall t\ge t_0.$$

We have

$$\begin{aligned}|(Fy)(t)|&\le Mg(t)|g^{-1}(t)X(t)X^{-1}(y_0)y_0|\\&+g(t)\int_{t_0}^{t}\|g^{-1}(t)X(t)X^{-1}(s)g(s)\|g^{-1}(s)\left|f\left(s,y(s),\max_{t_0\le\xi\le s}y(s)\right)-f(s,0,0)\right|ds\\&+g(t)\int_{t_0}^{t}\|g^{-1}(t)X(t)X^{-1}(s)g(s)\|g^{-1}(s)\left|f(s,0,0)\right|ds\\&\le g(t)Ky_0+g(t)K\int_{t_0}^{t}L_f(s)g^{-1}(s)\max\left\{|y(s)|,\left|\max_{t_0\le\xi\le s}y(\xi)\right|\right\}\\&+g(t)K\int_{t_0}^{t}|g^{-1}(s)f(s,0,0)|ds\\&\le g(t)Ky_0+g(t)K\|y\|_{BC_g}\int_{t_0}^{\infty}L_f(s)ds+g(t)K\int_{t_0}^{\infty}|g^{-1}(s)f(s,0,0)|ds,\end{aligned}$$

which implies

$$\sup_{t\ge t_0}\frac{|(Fy)(t)|}{g(t)}\le K\left(y_0+\|y\|_{BC_g}\int_{t_0}^{\infty}L_f(s)ds+\int_{t_0}^{\infty}|g^{-1}(s)f(s,0,0)|ds\right)<\infty.$$

Thus, $F:BC_g([t_0,\infty),\mathbb{R}^n)\to BC_g([t_0,\infty),\mathbb{R}^n)$.

Furthermore, for any $y_1,y_2\in BC_g([t_0,\infty),\mathbb{R}^n)$ one has

$$\begin{aligned}&|(Fy_1)(t)-(Fy_2)(t)|\\&=\left|X(t)X^{-1}(t_0)y_0+\int_{t_0}^{t}X(t)X^{-1}(s)f\left(s,y_1(s),\max_{t_0\le\xi\le s}y_1(\xi)\right)ds\right.\\&\qquad\left.-X(t)X^{-1}(a)y_0-\int_{t_0}^{t}X(t)X^{-1}(s)f\left(s,y_2(s),\max_{t_0\le\xi\le s}y_2(\xi)\right)ds\right|\\&\le g(t)\int_{t_0}^{t}\left\|g^{-1}(t)X(t)X^{-1}(s)g(s)\right\|L_f(s)g^{-1}(s)\max\left\{|y_1(s)-y_2(s)|,\left|\max_{t_0\le\xi\le s}y_1(\xi)-\max_{t_0\le\xi\le s}y_2(\xi)\right|\right\}\\&\le g(t)K\int_{t_0}^{t}L_f(s)g^{-1}(s)\max\left\{|y_1(s)-y_2(s)|,\max_{t_0\le\xi\le s}|y_1(\xi)-y_2(\xi)\right\}ds\\&\le g(t)K\int_{t_0}^{t}L_f(s)ds\|y_1,-y_2\|_{BC_g}.\end{aligned}$$

Here, we conclude that

$$\| Ty_1 - Ty_2 \|_{BC_g} \le K \int_{t_0}^{\infty} L_f(s)ds \cdot \| y_1, - y_2 \|_{BC_g}, \ \forall \, y_1, y_2 \in BC_g([t_0, \infty), \mathbb{R}^n).$$

By the condition d), T is a contraction mapping on $BC_g([t_0,\infty),\mathbb{R}^n)$. Thus, there exists a unique solution to (1.3) in $BC_g([t_0,\infty),\mathbb{R}^n)$ which is a fixed point of T.

3. C_g - Asymptotic equivalence

Definition 3.1 ([7]). Let $g(t)$ be a positive continuous function on $[t_0,\infty)$. We say that Equation (1.2) and (1.3) (or the system (1.4) - (1.5)) are C_g - equivalent on $[t_0,\infty)$ if and only if to each solution $x(\cdot,t_0)$ of (1.2) there exists a solution $y(\cdot,t_0)$ of (1.3) such that

$$\lim_{t\to\infty} \frac{| x(t,t_0) - y(t,t_0) |}{g(t)} = 0 \tag{3.1}$$

and conversely, for each solution y of (1.3) there exists a solution x of (1.2) such that (3.1) holds.

Theorem 3.2. Let $X(t)$ be a fundamental matrix of the system (1.2). We suppose that

a) the Assumption 1.2 holds;

b) $\int_{t_0}^{\infty} g^{-1}(s) \| f(s,0,0) \| ds < \infty$;

c) there exists two projectors P_1, P_2 of $\mathbb{R}^n$ and a constant $K > 0$ such that

$$\| g(t)X(t)P_1X^{-1}(s)g^{-1}(s) \| \le K, \text{ for } t_0 \le s \le t,$$
$$\| g(t)X(t)P_2X^{-1}(s)g^{-1}(s) \| \le K, \text{ for } t_0 \le t \le s,$$
$$\lim_{t\to\infty} g^{-1}(t)X(t)P_1 = 0;$$

d) $2K\int_{t_0}^{\infty} L_f(s)ds < 1$.

Then (1.2) and (1.3) are C_g - asymptotically equivalent.

Proof. Let x be a BC_g -solution of (1.2).

Corresponding to x we consider the operator

$$Ty(t) = x(t) + \int_{t_0}^{t} X(t)P_1X^{-1}(s) f\left(s, y(s), \max_{t_0\le\xi\le s} y(\xi)\right) ds -$$
$$- \int_{t}^{\infty} X(t)P_2X^{-1}(s) f\left(s, y(s), \max_{t_0\le\xi\le s} y(\xi)\right) ds.$$

We show that $BC_g\left([t_0,\infty),\mathbb{R}^n\right)$ is invariant for the operator T.

If $y \in BC_g\left([t_0,\infty),\mathbb{R}^n\right)$ then

$$\left| f\left(t, y(t), \max_{t_0\le\xi\le t} y(\xi)\right)\right| \le \left| f\left(t, y(t), \max_{t_0\le\xi\le t} y(\xi)\right) - f(t,0,0)\right| + | f(t,0,0)|$$

$$\le L_f(t)\max\left(| y(t), \left|\max_{\xi\in[t_0,t]} y(\xi)\right|\right) + | f(t,0,0)|$$

$$\le L_f(t)\| y\| + | f(t,0,0)|.$$

Let x be a BC_g - solution of (1.2) and $y \in BC_g\left([t_0,\infty),\mathbb{R}^n\right)$. Then

$$| Ty(t)| \le Mg(t) + g(t)\int_{t_0}^{t}\left| g^{-1}(t)X(t)P_1X^{-1}(s)g(s)g^{-1}(s) f\left(t, y(t), \max_{t_0\le\xi\le t} y(\xi)\right)\right| ds$$

$$+ g(t)\int_{t}^{\infty}\left| g^{-1}(t)X(t)P_2X^{-1}(s)g(s)g^{-1}(s) f\left(s, y(s), \max_{t_0\le\xi\le s} y(\xi)\right)\right| ds$$

$$\le Mg(t) + g(t)K\| y\|_{BC_g}\int_{t_0}^{t} L_f(s)ds + g(t)K\int_{t_0}^{t} | g^{-1}(s) f(s,0,0)| ds$$

$$+ g(t)K\| y\|_{BC_g}\int_{t}^{\infty} L_f(s)ds + g(t)K\int_{t_0}^{\infty} g^{-1}(s)| f(s,0,0)| ds.$$

So

$$\| Ty\|_{BC_g} \le M + K\| y\|_{BC_g}\int_{t_0}^{\infty} L_f(s)ds + K\int_{t_0}^{\infty} g^{-1}(s)| f(s,0,0)| ds < \infty.$$

Now we prove that T is a contraction on $BC_g\left([t_0,\infty),\mathbb{R}^n\right)$

$$| Ty_1(t) - Ty_2(t)|$$

$$\le g(t)\int_{t_0}^{t}| g^{-1}(t)X(t)P_1X^{-1}(s)g(s)| g^{-1}(s)\left| f\left(s, y_1(s), \max_{t_0\le\xi\le s} y_1(\xi)\right) - f\left(s, y_2(s), \max_{t_0\le\xi\le s} y_2(\xi)\right)\right|$$

$$+ g(t)\int_{t}^{\infty}| g^{-1}(t)X(t)P_2X^{-1}(s)g(s)| g^{-1}(s)\left| f\left(s, y_1(s), \max_{t_0\le\xi\le s} y_1(\xi)\right) - f\left(s, y_2(s), \max_{t_0\le\xi\le s} y_2(\xi)\right)\right|$$

$$\le g(t)K\int_{t_0}^{t} L_f(s)g^{-1}(s)\max\left(| y_1(s) - y_2(s), \left|\max_{t_0\le\xi\le s} y_1(s) - \max_{t_0\le\xi\le s} y_2(s)\right|\right) ds$$

$$+ g(t)K\int_{t}^{\infty} L_f(s)g^{-1}(s)\max\left(| y_1(s) - y_2(s), \left|\max_{t_0\le\xi\le s} y_1(s) - \max_{t_0\le\xi\le s} y_2(s)\right|\right) ds$$

$$\le g(t)K\| y_1 - y_2\|_{BC_g}\int_{t_0}^{t} L_f(s)ds + g(t)K\| y_1 - y_2\|_{BC_g}\int_{t}^{\infty} L_f(s)ds,\ \ \forall y_1, y_2 \in BC_g\left([t_0,\infty),\mathbb{R}^n\right).$$

Therefore,

$$\| Ty_1 - Ty_2\|_{BC_g} \le \left(2K\int_{t_0}^{\infty} L_f(s)ds\right)\| y_1 - y_2\|_{BC_g} \quad \forall y_1, y_2 \in BC_g\left([t_0,\infty),\mathbb{R}^n\right).$$

By Banach's fixed point theorem, there exists a unique solution of (1.3). Let y be a solution of (1.3) corresponding to x. Then

$$\begin{aligned}
&|x(t)-y(t)| \\
&\le g(t)\int_{t_0}^{t}\left|g^{-1}(t)X(t)P_1X^{-1}g(s)g^{-1}(s)f\left(s,y_1(s),\max_{t_0\le\xi\le s}y_1(\xi)\right)\right|ds \\
&\quad+g(t)\int_{t}^{\infty}\left|g^{-1}(t)X(t)P_2X^{-1}(s)g(s)g^{-1}(s)f\left(s,y_1(s),\max_{t_0\le\xi\le s}y_1(\xi)\right)\right|ds \\
&\le g(t)\int_{t_0}^{t_1}\left|g^{-1}(t)X(t)P_1X^{-1}g(s)g^{-1}(s)f\left(s,y_1(s),\max_{t_0\le\xi\le s}y_1(\xi)\right)\right|ds \\
&\quad+g(t)\int_{t_1}^{t}\left|g^{-1}(t)X(t)P_1X^{-1}(s)g(s)g^{-1}(s)f\left(s,y_1(s),\max_{t_0\le\xi\le s}y_1(\xi)\right)\right|ds \\
&\quad+g(t)\int_{t}^{\infty}\left|g^{-1}(t)X(t)P_2X^{-1}(s)g(s)g^{-1}(s)f\left(s,y_1(s),\max_{t_0\le\xi\le s}y_1(\xi)\right)\right|ds.
\end{aligned}$$

Hence

$$\|x-y\|_{BC_g}|X(t)P_1|\int_{t_0}^{t_1}|X^{-1}(s)|\left|f\left(s,y(s),\max_{t_0\le\xi\le s}y(\xi)\right)\right|ds+K\int_{t_1}^{\infty}\left|f\left(s,y(s),\max_{t_0\le\xi\le s}y(\xi)\right)\right|ds\,.$$

We fix $\varepsilon>0$ and determine $t_1\ge t_0$ such that the second integral in the right-hand side of the above inequality is less than $\varepsilon/2$. With t_1 fixed, from hypothesis c), we have that the first term on the right-hand side of the above inequality tends to zero as \$x\$ tends to infinity. We may therefore conclude that $\lim_{t\to\infty}\frac{\|x(t)-y(t)\|}{g(t)}=0.$

Similarly, we take a bounded solution y of (1.3) and we observe that

$$\begin{aligned}
x(t)=y(t)-\int_{t_0}^{t}X(t)P_1X^{-1}(s)f\left(s,y(s),\max_{t_0\le\xi\le s}y(\xi)\right)ds \\
+\int_{t}^{\infty}X(t)P_2X^{-1}(s)f\left(s,y(s),\max_{t_0\le\xi\le s}y(\xi)\right)ds
\end{aligned}$$

satisfies (1.2) and (1.6) holds.

Remark 3.3. Note that, when $g\equiv 1$ we obtain the result which is presented by Otrocol [8] as mentioned before.

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APPLYING GOOGLE CLASSROOM AS A BLENDED TOOL FOR THE DEVELOPMENT OF THE LEARNING MODEL OF ENGLISH WRITING COURSE FOR ENGLISH-MAJOR STUDENTS AT HONG DUC UNIVERSITY

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Abstract: *Applying technology in teaching and learning is inevitable in this current era of rapid development of science and technology. This study is aimed at exploring the application of Google Classroom as a blended tool for the development of the learning model of English writing course for English-major students at Hong Duc University. A survey questionnaire on students' experience of Google Classroom and an analysis of two paragraphs that students wrote were used as the data sources. The results showed the students' positive responses to the program and their considerable improvement in writing skill. The further studies on developing online classroom's materials resources as well as on the effectiveness of using Google Classroom in English teaching and learning need to be continued.*

Keywords: *Google Classroom, writing skill, technology in teaching and learning, tool for the development of the learning model.*

1. Introduction

In the current era of rapid development of science and technology, teaching at college following traditional methods cannot fully meet the learners' needs as well as the increasing demands of the society. Recently, blended learning, which was defined as a combination of "online and face-to-face course delivery" [2, p.137], has been increasingly becoming popular and trendy in the field of academic area in general and in foreign language teaching in particular. It is said to take advantages and make good the shortcomings of both face-to-face and online learning to meet "learners' expectation", "learners' flexibility" and "authority's directives" [3, p.58]. Among a broad range of different methods to apply blended learning, using Google Classroom along with face-to-face classroom has recently been widely used and highly recommended by a lot of teachers throughout the world [4] [6] [7] [8] [10].

In Vietnamese context, many materials and seminars as well as workshops have been frequently given to guide both teachers and students to exploit this application to make students' self-study productive and increase learning effectiveness in language teaching and learning programs [1]. The similar picture is captured at Hong Duc University, where all the language teaching and learning classrooms are well-equipped with computers,

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projectors, loud-speakers a long with Wi-Fi systems. Moreover, the content of English Language Skills Development courses in the English Language Teachers Training Program and English Language Program at Hong Duc University is compatible with using Google Classroom as an effective tool to support teacher-student-material interaction to develop students' autonomy and self-assessment. This seems to be more evitable during the current COVID-19 pandemic period.

This paper, therefore, is aimed at investigating the use of Google Classroom as a blended tool for the development of the learning model of English writing course for English-major students at Hong Duc University. The students' attitude towards Google Classroom as well as the effectiveness of using this platform along with traditional lectures to provide materials and give feedback to students' weekly home assignments uploaded are evaluated after finishing the semester.

2. Literature review

2.1. Introduction of Google Classroom

Google Classroom is a free web service first launched by Google on May 6, 2014 and officially announced on August 12, 2014 [10]. This is an education-friendly platform, which is integrated with other Google services such as Google Drive, Google Docs, Google Sheets, Google Slides, etc. For teachers and students, Google Classroom brings the benefits of paperless sharing and digital collaboration to classrooms. Tens of millions of teachers and students have used Google Classroom in thousands of schools around the world, making it one of the most popular edtech tools around [5]. The three most important features of Google Classroom are communication, assignment and storage. Learners can join a class with a class code provided by teachers or automatically added by the school administration. After joining the class, a folder named Google Classroom is created by default in that learner's Drive to store all the online assignments that the learner submits to the teacher. Teachers can track the learning progress, mark papers, give comments as well as check the learners' ratings. In addition to the web version, Google Classroom is also available for mobile applications using iOS or Android operating systems [11].

2.2. Advantages of Google Classroom in teaching and learning

Applying Google Classroom in teaching and learning is reported to be considerably advantageous. It helps both teachers and learners organize and manage teaching and learning activities more easily and conveniently without taking too much time and money [4] [8] [10]. That all the documents, assignments and grades are stored in the same place in Google Drive makes it easier for teachers to manage students' participation and assess their learning progress. Likewise, learners do not have to use too much paper when doing exercises and do not have to worry about losing the assignments that they have done before.

In addition, the ability to synchronize with other Google applications is perfectly integrated, providing optimal convenience for users. The variety of web and mobile versions also allows teachers and learners to actively access the classroom quickly and conveniently at any time and in any places. Moreover, the school's Google account has been granted with unlimited capacity, which allow teachers to store all teaching materials, reference videos, class photos, grades, and etc... right on the Drive of this classroom and share it with learners without having to worry about storage space. This is obviously a remarkably effective platform for teachers and students to apply modern teaching methods using technology in teaching in order to catch up with the trend of the industrial revolution 4.0.

3. Methodology

3.1. Participants

The study was conducted with 64 participants studying the course of "Reading and writing 2" at Hong Duc University. Among them, 36 students come from class K23A and 28 others come from class K23B. All of them are English-major freshmen studying English Language Program. The course lasted for 13 weeks from March to June of 2021. This course is for students at upper A2 level under the same curriculum using Google Classroom.

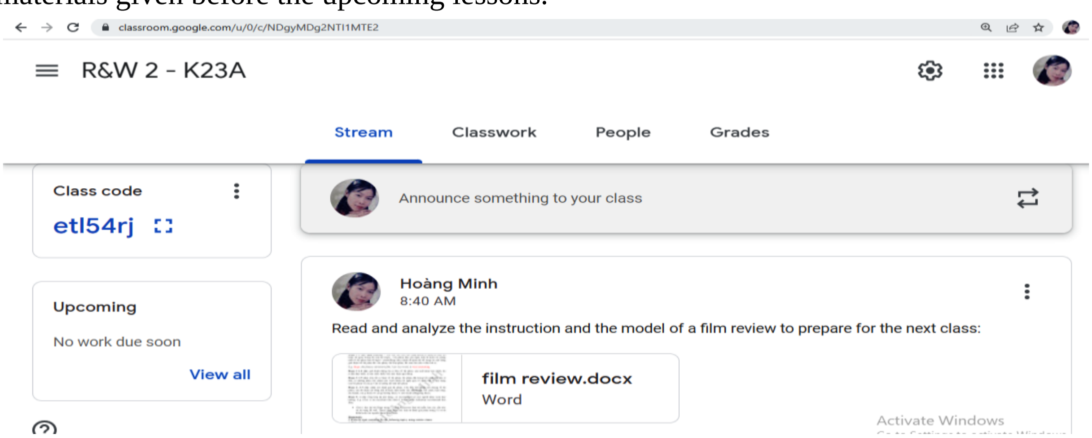
3.2. Procedure of the study

The "Reading and writing 2" course use Richmond's "Achievers B1" and "Effective Academic Writing 1" as the main textbooks.

The teacher applied Google Classroom as a tool for the development of the learning model of "Reading and writing 2" course with several activities:

Creating classes and adding students to the classes

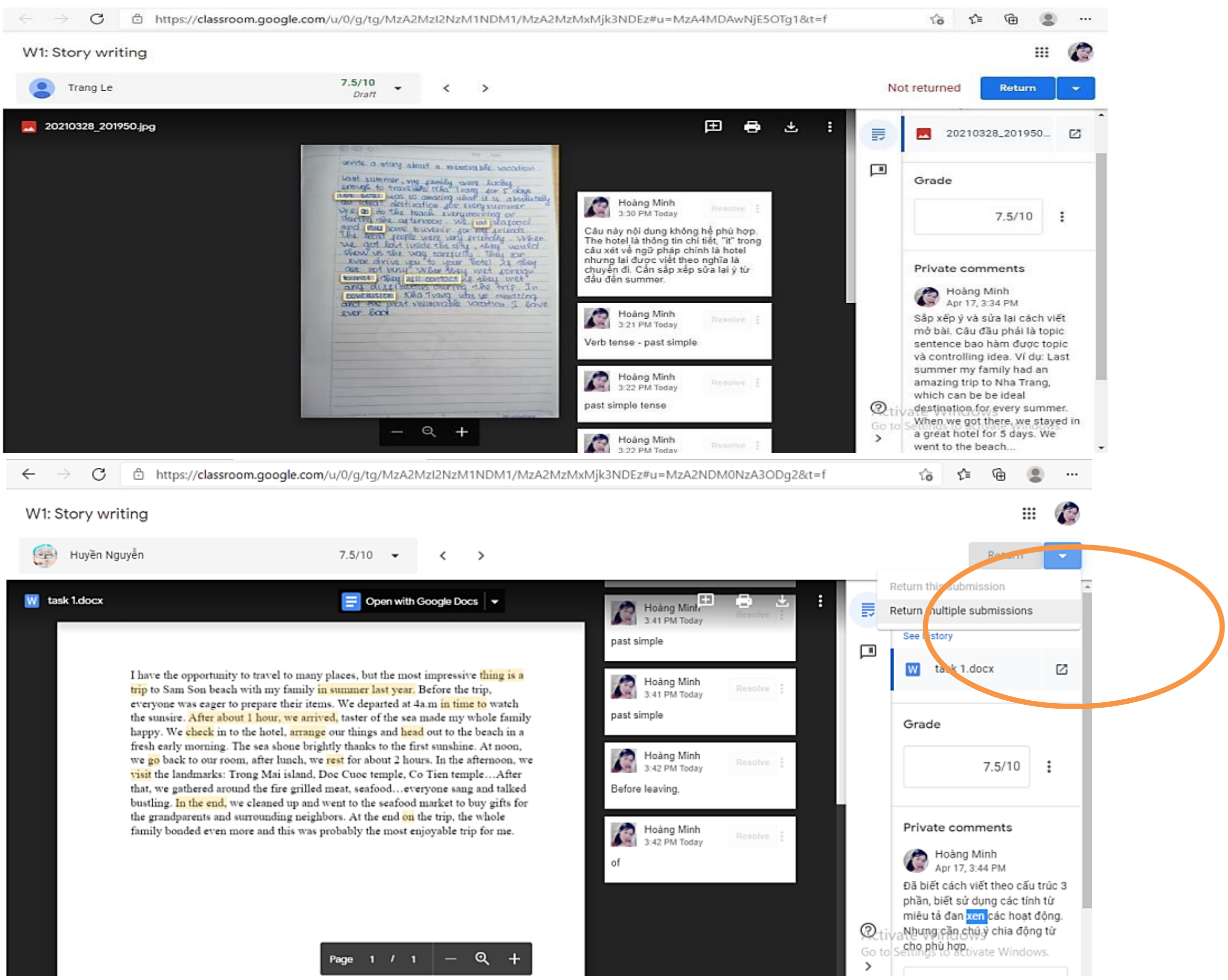
Providing materials (Picture 1): On the "*Stream*" page, the teacher uploaded the curriculum, the syllabus, the PowerPoint lessons, the videos lectures, announcements, instructions and any other materials that needs to provide for students. All members of the class are required to post comments about their understandings and questions about the materials given before the upcoming lessons.



Picture 1. *An example of researcher's class materials uploaded on Google Classroom*

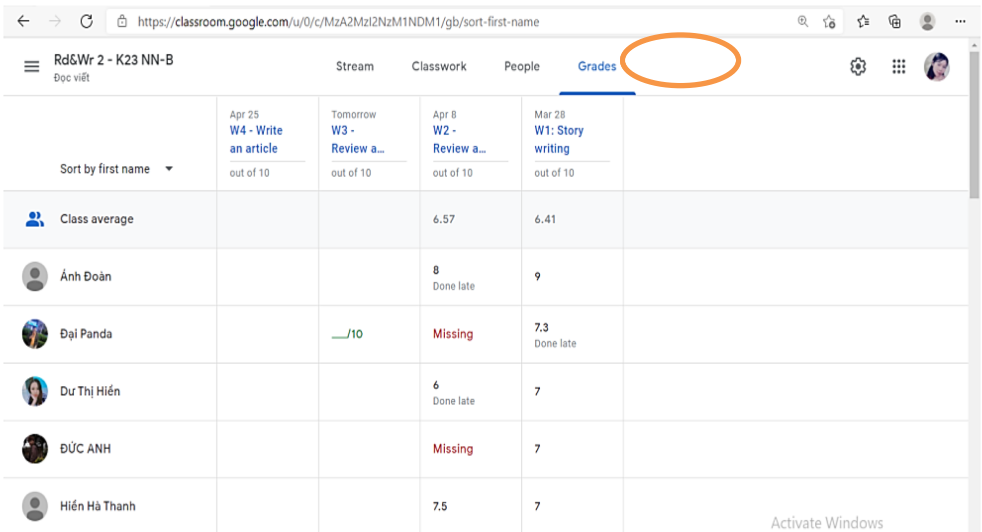
Assigning assignments: On the "Classwork" page, the teachers created different assignments and projects for students and set the due date for each assignment. The assignments were varied with free writings, instructed writings, live worksheets, oral presentations of the writing… With instructed writings, the students were required to write and upload 7 writings including a story, an article, a review and four paragraphs of four different kinds (example, process, opinion and narrative paragraphs) which were to be examined in the final test**.**

Grading and giving feedback (Picture 2): After the students completed and turned in their work, the teacher graded and returned it to the students. The system will filter and sort separately the group of members who submitted their papers on time, groups of members who submitted their papers late, groups of members who have not submitted their papers, and groups of graded members. Teachers gave students personalized feedback in the Classroom grading tool, left comments on student work and maintained a comment bank to store comments used most often.



Picture 2. *An example of researcher's feedback giving on Google Classroom*

Checking students' participation and grades (Picture 3): The teacher entered the "grades" section. Here the application has systematically sorted out the assignments in chronological order and reported the results of each student's grades as well as the point average of each assignment. Moreover, the application also clearly reports which members submitted their entries on time, which submitted their papers late, which have not submitted their papers, and which have been graded. Therefore, the teacher can easily control and evaluate the participation and learning progress.



Picture 3. *An example of the report of participants' grades on Google Classroom*

Some other activities: In addition to the teaching and managing activities in each class, teachers can also keep track of all classes' work due dates in "*calendar*", all work in progress in "*to review*", archived classes that were done teaching in "*archived classes*" so as to have an appropriate assessment and adjustment if needed. As a part of regular progress test, students are asked to write an example paragraph in week 12, which is similar in type to the one they had done on Google Classroom in week 7. These two pieces of essays were analyzed to investigate the improvement of students' writing skills.

Furthermore, at the end of the semester, an online survey was conducted via Google Form to collect students' feedback regarding the application of Google Classroom as an online learning management system. The survey questionnaire involved 10 questions with 9 multiple-choice and 1 open-ended question. The 9 multiple-choice questions are aimed at gaining students' perception of the familiarity and user-friendliness of Google Classroom, the frequency and the reasons they visited Google Classroom, their opinion about the most interesting and useful activity on Google Classroom, their habits of turning in assignments and their willingness to use Google Classroom in the later courses. The open-ended question deals with the difficulties students encountered while using Google Classroom in the course.

3.3. Data analysis

For the analysis of survey data, the study employs both quantitative and qualitative analysis methods.

For the analysis of survey data, the study employs both quantitative and qualitative analysis methods. Concerning quantitative data from multiple choice questions used in the questionnaire, the researcher used descriptive statistics by calculating and comparing the percentage of students choosing different options to comprehensively evaluate the students' experience in using Google Classroom. The quantitative and qualitative data of students' essays were also analyzed and assessed based on the scoring scheme used in the curriculum.

4. Findings and Discussion

4.1. Students' perception of using Google Classroom in learning

When being surveyed, all sixty-four participants sent back their responses. The researcher expected to get only one answer for each question, but a few questions received multiple answers and the open-ended question went unanswered in some responses.

The first question was about the familiarity with Google Classroom. 64/64 students indicated that they had never used Google Classroom in any courses before. That means all the Google Classroom features are completely new to them.

When being asked about the features of Google Classroom, 51% participants affirmed that they are simple and user-friendly, 44% said they are average while only 2% thought they are very simple, 3% stated they are complex and no one considered them very complex. This result shows that the features of Google Classroom themselves hardly cause any problems for students to use.

Answering the third question about the frequency the students visited Google Classroom, majority of them asserted they assessed the platform 2 or 3 times a week (72%). Meanwhile, 17% visited Google Classroom once or less than one a week, 11% visited the site 4 or 5 times a week and no students visited it more frequently. This is a quite positive result because the instructions and assignments were posted weekly before the classes, which took place twice a week. It also indicates that students are pretty serious in combining classwork and online materials with assignments given.

Question 4 about the most common reasons for visiting Google Classroom was the one receiving multi-select answers (Table 1).

Table 1. *Reasons students visited Google Classroom*

Options	Number of students selected	Percentage of students selected
Reading curriculum and syllabus	0	0%
Watching video lectures or PowerPoint lessons	25	39%
Checking instructions and announcements	35	55%
Uploading homework assignments	64	100%
Checking teacher's writing feedbacks	64	100%

It is obvious that their main purpose in using Google Classroom was for turning in the homework and checking the feedbacks which they are required to do weekly. Many students also pay attention to extra materials like instructions and announcements or video lectures and PowerPoint lessons, but no one care for the curriculum and syllabus provided online.

Question 5 and 6 inquired about students' opinion about the most interesting and useful activity on Google Classroom (Table 2). These questions also got multi-select answers. It is not very strange that all students appreciate the importance and usefulness of checking teacher's writing feedbacks but only 34% of them considered it interesting. On the contrary, a high percentage of learners liked to watch video lectures or PowerPoint lessons and recording oral presentation of their writings (50% and 66% respectively) but only a small number of students think these activities are beneficial. The most dominantly, no students are fond of reading curriculum and syllabus.

Table 2. *Students' opinion about the most interesting and useful activity on Google Classroom*

Options	The rate of students selected as the most interesting activity	The rate of students selected as the most useful activity
Reading curriculum and syllabus	0 (0%)	0 (0%)
Watching video lectures or PowerPoint lessons	32 (50%)	21 (33%)
Checking instructions and announcements	2 (3%)	20 (31%)
Writing and uploading written assignments	5 (8%)	45 (70%)
Recording oral presentations of the writing	42 (66%)	12 (19%)
Checking teacher's writing feedbacks	22 (34%)	64 (100%)

Question 7 and 8 referred to students' habits of turning in assignments. More than two-thirds of them usually turned in home assignments on time while only 13% chose "always", 12% chose "often" and the rest chose "sometimes". No students said they rarely turned in the assignment before the due date. Optimistically, 75% of them did all 7 out of 7 written work assigned, 2/64 (3%) uploaded more than 7 assignments as required and the rest only missed 1 or 2 assignments. This fact indicates students' positive and serious attitude in doing and uploading homework on Google Classroom.

However, there were still some problems with the use of Google Classroom as a tool in managing teaching and learning process. In open-ended question number 9 about the difficulties encountered while using Google Classroom, students listed some of the problems as followed:

Internet connection was not always stable, causing a lot of annoyance.

Sometime students could not attach files.

Sometimes the system did not report that the students' writings were turned in although they had already been submitted and the teachers could still see and open the files to give feedback as normal.

Students could not see others' assignments and feedbacks to learn from peers' strengths and weaknesses.

They also noted that these problems are not frequent and hoped Google Classroom could solve them.

Noticeably, in the last question referring to the students' wishes to continue to use Google Classroom as a blended tool for the development of the learning model, all 64 out of 64 students confirmed that they definitely want to work with this platform for the next courses.

In conclusion, most students express their positive attitude and experience towards using Google Classroom in studying writing. They also hold expectation to tackle all the difficulties caused during the process of teaching and learning.

4.2. Analysis of the two writing papers

The two writing papers are in the same type of an example paragraph examined in week 7 on Google Classroom and week 12 in the progress test. The first writing topic is 'one important quality of a good friend' and the week 12 test's topic is "one important quality of a good teacher". They are assessed based on an analytic scoring rubric with four criteria: task fulfillment, organization, vocabulary and grammar. The results were really positive with a considerable improvement in most students (Table 3).

Table 3. *Students' writing marks in week 7 and week 12*

Score	≤ 4	5-6	6.5-7.5	8-8.5	≥ 9
Week 7	4	22	26	8	4
Week 12	0	12	25	19	8

It is a good sign that the number of students getting mark 8 or above doubled from nearly 20% to more than 40% while there were no more students getting mark 4 or below. It is also worth noting that the figure of students getting mark 5-6 decreased from 34% to 19%. The number of students getting mark 6.5-7.5 seemed to stay the same, but most of them are the one who had got lower marks before. Another interesting point should be mentioned is that only 15% (10/64) students got the same mark in two writings whereas all others got higher mark.

In term of the specific criteria in the writings, the researcher found that the most noticeable improvement in students' writing is the way they organized the ideas, elaborated their ideas with relevant explanations as well as examples using more appropriate linking devices. In the first test, 8 out of 64 students misunderstood the question by writing about several qualities of a good friend but not one as required. This situation was not repeated in the final test. Furthermore, majority of students still write a lot of unrelated sentences, especially in the opening of the paragraph in the first writing, and just more than one third students could write a paragraph in the expected form with a good topic sentence, a concluding sentence and a range of quite relevant supporting ideas. This figure increased to more than two thirds in the final test in week 12. More noticeably, the way they supported and summarized ideas was much more relevant with better collaborative explanations and examples. All 64 students in these two classes also showed their improvement in using transitional words devices to connect the ideas in the final test. In the former test, most of the conjunctions used are basic like ‘first’, ‘second’, ‘third’, ‘and’, ‘so’, ‘but’... However, the teacher found a variety of cohesive devices such as ‘the most important quality is that’, ‘furthermore’, ‘moreover’, ‘for instance’, ‘an example of…is…’ in the later one. In addition, they also used broader range of vocabulary and grammar with higher degree of accuracy. The number of complex and compound sentences appeared in the second writing was much more frequent. This was the results of their frequency in writing with the assistance of the instruction and feedback given on Google Classroom.

To sum up, all those positive results presented that students’ writing skills improved significantly from the beginning to the end of the course.

3. Conclusion

Using Google Classroom as a blended tool for the development of the learning model in teaching English writing for English-major students at Hong Duc University is preliminarily assessed to be useful and effective to enhance students’ engagement and improve their writing skills. All the modules in Reading and Writing Skills can be compatible with Google Classroom’s features to help students get acquainted and use the web environment skillfully, learn to share learning databases and links to digital libraries, improve collaborating skills with peers and teachers. The blended learning of online classes with face-to-face classes will give students many opportunities to develop their learning autonomy with their classmates’ and teachers’ assistance. The future research direction is to build enrichment resources for Google Classroom, especially the Quiz / Assignment bank and to apply Google Classroom to other subjects. More importantly, more studies on the effectiveness of this application under different objective aspects from both teachers and learners need to be conducted in order to really improve the quality of teaching and learning at university.

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AN INVESTIGATION OF VOCABULARY LEARNING STRATEGIES USED BY NON-ENGLISH MAJORS AT HONG DUC UNIVERSITY

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Abstract: *This article aims at studying the English vocabulary learning strategies (VLSs) employed by non-English majors at Hong Duc University (HDU) and the frequencies of the strategies used. A set of questionnaires adapted from Schmitt's taxonomy of VLSs was delivered to 140 non-English majored students as a tool for data collection. The results from data analysis reveal some major findings about frequency of students' reported overall VLSs use and that of the six main strategy categories, the most frequently used VLSs and the least frequently used VLSs. Based on these findings, the writer has provided some recommendations with the hope of improving English vocabulary teaching and learning at HDU.*

Keywords: *Vocabulary learning strategies (VLSs), frequency of students' VLSs use, Schmitt's Taxonomy of Vocabulary Learning Strategies*.

1. Introduction

Good learning strategies may have a great contribution to the learners' success in language learning. Chamot, Barnhardt, El-Dinary and Robbins [4] stated: "differences between more effective learners and less effective learners were found in the number and range of strategies used, in how the strategies were applied to the task, and in whether they were appropriate for the task". It can be inferred that with appropriate learning strategies, students can learn faster and more effectively. In addition, developing the students' knowledge of English cannot be separated from vocabulary mastery. It is obvious that vocabulary is an important part of linguistic knowledge. Zimmerman [16] indicates: "Vocabulary is central to language and of crucial importance to the typical language learner". Fromkin et. al. [9] also states: "Knowing a language means knowing the words of that language". The more words we know, the better we can convey our thoughts. Consequently, vocabulary acquisition has always been a fundamental and important activity in English learning and teaching.

The aforementioned important roles of learning strategies and vocabulary learning are the main reasons which prompt the writer to conduct this study to get an insight into this matter. Besides, the ideas for this study also come from the researcher's own experience as an English teacher for non-English majors at HDU. As a teacher, the writer can recognize the deficiency in non-English majored students' vocabulary knowledge, which in turn has a bad influence on their English learning process and achievement. This research, therefore, sets its objectives to investigate the Vocabulary learning strategies (VLSs) used by HDU non-English majors and frequency of students' vocabulary learning strategy use. Based on this background knowledge, the researcher gives some implications to vocabulary English teaching and learning at HDU.

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2. Theoretical background

2.1. Definitions of Vocabulary Learning Strategies

To generally define VLSs, Nation [12, p.217] states that VLSs are a subclass of language learning strategies. He claims that it is not easy to define what a strategy is, but “a strategy would need to: (1) involve choice, that is, there are several strategies to choose from, (2) be complex, that is, there are several steps to learn, (3) require knowledge and benefit from training, and (4) increase the efficiency of vocabulary learning and vocabulary use” [8, p.217].

Cameron [2] defines VLSs as “actions that learners take to help themselves understand and remember vocabulary”. (Cameron 2001, p.92; as cited in Fooziyeh Rasouli & Khadijeh Jafari 2016). Similarly, Catalan [3, p.56] defines VLSs as “knowledge about the mechanisms (processes, strategies) used in order to learn vocabulary as well as steps or actions taken by students (a) to find out the meaning of unknown words, (b) to retain them in long-term memory, and (c) to recall them at will, and (d) to use them in oral or written mode”.

In general, all definitions mentioned above all share the same goals to help learners learn and use vocabulary more easily and effectively. They are the actions and techniques which learners use to help them discover the meaning of a new word, remember it, retrieve it when necessary and increase their vocabulary knowledge.

2.2. Schmitt’s Taxonomy of Vocabulary Learning Strategies

To briefly describe his classification, Schmitt [15, p.206-207] affirms in his research that it is organized according to both the Oxford [13] system and the Discovery/Consolidation distinction suggested by Cook and Mayer [7]. In other words, this taxonomy takes advantage of Oxford’s categories including Social strategies (SOC), Memory strategies (MEM), Cognitive strategies (COG), and Metacognitive strategies (MET). However, his classification differs from Oxford’s in terms of the presence of a new category - Determination Strategies (DET).

Schmitt’s taxonomy includes two major classes, namely, strategies for the discovery of a new word’s meaning and strategies for consolidating a word once it has been encountered. Each class is divided into different sets of strategies. Totally, his present taxonomy contains 58 strategies.

2.2.1. Strategies for the discovery of a new word’s meaning

These strategies are used to discover initial information about a new word when learners encounter it for the first time. This class includes Determination strategies and Social strategies.

Determination strategies (DET): help learners discover the meaning of a new word by using four options: guessing from their structural knowledge of the language, guessing from an L1 cognate, guessing from context, using reference materials.

Social strategies (SOC): help learners discover the meaning of a new word by asking someone who knows. Learners can ask their teacher or classmates for L1 translation, paraphrase, synonym, a model sentence containing that word; or they can discover the meaning through group work activity.

2.2.2. Strategies for consolidating a word once it has been encountered

Once learners have gained initial information about a new word, they need to use some strategies to remember it. These strategies are called Consolidation strategies which contain Social SOC), Memory (MEM), Cognitive (COG) and Metacognitive strategies (MET).

Social strategies (SOC): Besides their use in the discovery of the meaning of a new word, **s**ocial strategies can also be employed to learn or practice vocabulary. Learners can work in a group or interact with native-speakers. This group also involves teacher's checking students' flashcards word lists for accuracy.

Memory strategies (MEM): (traditionally known as mnemonics) are used to retain the word, using some form of imagery, or grouping. A new word "can be integrated into many kinds of existing knowledge (i.e. previous experiences or known words) or images can be custom-made for retrieval (i.e. images of the word's form or meaning attributes)" [4, p.211].

Cognitive strategies (COG): "are similar to memory strategies, but are not focused so specifically on manipulative mental processing" [4, p.219]. Cognitive strategies contain repetition and using mechanical means to study vocabulary. Learners can write or say a word again and again to remember it. Besides, they can take notes in class, make use of special vocabulary sections in their textbooks, study by listening, record a tape of word lists or label their physical objects in L2.

Metacognitive strategies (MET): are used by students "to control and evaluate their learning, by having an overview of the learning process in general" [4, p.220]. In the other words, metacognitive is related to planning and self-evaluation to reflect on the learning processes.

3. Setting of the study

The study was conducted at Hong Duc University (HDU) during the second semester of the academic year 2020-2021. At HDU, students are required to learn three English subjects with three respective levels, namely, English 1 (Elementary level), English 2 (Intermediate level) and English 3 (Advanced level). However, it is worth noticing that the general English level of students at HDU is not very high. Each semester, the number of students who do not pass English subjects is quite high. To some students, English is a real obstacle to get their bachelor's degrees. In addition, vocabulary is also one of the biggest problems to students in learning English.

The participants of this study were 140 non-English majors at HDU (77 females and 63 males). They were students of four different faculties, namely, Faculty of Economics, Faculty of Science and Technology, Faculty of Information Technology and Faculty of Primary Education. All of them have learnt English for seven years or more. These students were chosen randomly for the survey. They were taking part in English classes at the time of the survey. By investigating their ways of learning vocabulary and their use of VLSs, this study is hoped to draw some implications and suggestions for English vocabulary teaching and learning at HDU.

4. Data collection and analysis procedure

In this research, a set of questionnaire was used as two tools for data collection. The questionnaires were distributed to 140 non-English majored students to investigate the VLSs reported using by learners and the frequency of use of these strategies. Data collected from the questionnaires were analyzed quantitatively via Excel software for mean scores. In addition, descriptive statistics were also counted for percentage and ranks.

The mean scores were used to determine the frequency use of strategies by students. The means cores were interpreted as follows: In questionnaires, the frequency of strategy use was presented on a five-point rating scale, ranging from 'never or almost never', valued as 1; 'generally not used', valued as 2; 'sometimes used', valued as 3; 'usually used', valued as 4; and 'always or almost always used', valued as 5. This study is based on the Reference of mean score proposed by Oxford [13] in her SILL (Strategy Inventory for Language Learning) Profile of results version 7 to interpret the mean scores. Accordingly, the mean score of strategy use of each category or item which valued from 1.0 to 2.4 was determined as 'low use', from 2.5 to 3.4 as 'medium use', and from 3.5 to 5.0 as 'high use'.

Table 1. *Reference to mean scorebased on Oxford's (1990) SILL Profile of results version 7*

High	Always or almost always used Usually used	4.5 to 5.0 3.5 to 4.4
Medium	Sometimes used	2.5 to 3.4
Low	Generally not used Never or almost never used	1.5 to 2.4 1.0 to 1.4

5. Frequency of students' Vocabulary Learning Strategies use

5.1. Frequency of students' reported overall Vocabulary Learning Strategies use

Table 2 below reveals the mean score of overall 58 vocabulary strategies items in the questionnaires administered to 140 non-English majored students at HDU.

Table 2. *Frequency of students' overall vocabulary learning strategies use*

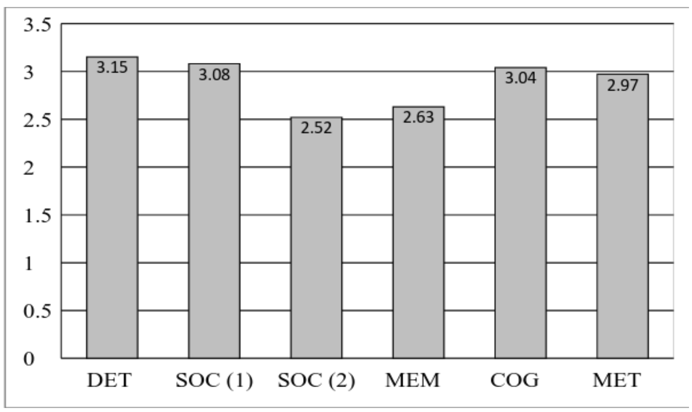
Students' Strategies Use	Mean Score	Frequency Category
Overall Strategy Use	2.84	Medium use
Strategies to discover the meaning of an English word	3.13	Medium use
Strategies to learn and practice an English word after finding out its meaning	2.74	Medium use

As can be seen in Table 2, the mean score of overall vocabulary learning strategies use reported by participants is 2.84. This means that participants of this research reported employing vocabulary learning strategies with medium frequency. In addition, the mean scores for the two sub-classes, namely Strategies to discover the meaning of an English word and Strategies to learn and practice an English word after finding out its meaning, are 3.13 and 2.74 respectively. They also belong to the medium frequency use category. However, it can be seen that the mean score of the former sub-class is bigger than that of the latter. Therefore, it can be concluded that 140 participants of this research used more strategies to discover meaning of an English word than to consolidate a new word.

5.2. Frequency of Use of Vocabulary Learning Strategies in the Six Main Strategy Categories

Chart 1 below reveals the mean scores of the six main strategy categories: Determination (DET), Social (SOC1) (for the discovery of a new word's meaning), Social (SOC2) (Consolidation strategies), Cognitive (COG), Memory (MEM) and Metacognitive (MET).

Chart 1. *The use of 6 strategy categories by non-English majors*



As can be seen from the above chart, the mean scores for six strategy categories range from 2.52 to 3.15. This means that 140 participants reported using VLSs in all six main categories at medium frequency. The ranking of mean scores for 6 strategy categories is presented in the following table.

Table 3. *Ranking of mean scores for 6 strategy categories*

Ranking	Strategy Categories	Mean score	Frequency category
1st	Determination	3.15	Medium use
2nd	Social (for the discovery of a new word's meaning)	3.08	Medium use
3rd	Cognitive	3.04	Medium use
4th	Metacognitive	2.97	Medium use
5th	Memory	2.63	Medium use
6th	Social (Consolidation strategies)	2.52	Medium use

This table shows that students used Determination category the most frequently with the mean score of 3.15. Social category (SOC) for the discovery of a new word's meaning ranks the second with the mean score quite closed to that of Determination category ($\overline{X}$= 3.08). It is followed by Cognitive (COG), Metacognitive (MET) and Memory (MEM) categories with the mean scores of 3.04, 2.97 and 2.63 respectively. Social (SOC) group for consolidating a new word ranks the last, but its mean score still belongs to medium frequency category ($\overline{X}$= 2.52).

5.3. The most frequently used Vocabulary Learning Strategies

Table 4 below presents the most frequently used VLSs, which are also 9 strategies which were reported using at high level of frequency.

Table 4. *Top frequently used vocabulary learning strategies*

Ranking	**Strategy categories**	**Item No.**	**Vocabulary learning strategies**	**Mean score**
1st	DET	6	I use a bilingual English/Vietnamese dictionary.	4.94
2nd	COG	45	I use verbal repetition.	4.3
3rd	MEM	31	I study the spelling of a word.	4.26
4th	COG	46	I use written repetition.	4.25
5th	MEM	32	I study the sound of a word.	4.23
6th	MEM	33	I say the word aloud when studying.	3.92
7th	MET	58	I continue to study the new English word overtime.	3.74
8th	SOC	10	I ask the teacher for the Vietnamese translation of the word.	3.68
9th	SOC	13	I ask my classmates for the Vietnamese meaning of the word.	3.58

As demonstrated in the above table, the 9 most frequently used VLSs belong to Determination, Social (for the discovery of a new word's meaning), Cognitive, Memory and Metacognitive groups. This means that no Social strategy for the consolidation of a new word was used at high frequency level. This result completely coincides with the data presented in table 3 when Social group (consolidation strategies) ranks the last amongst 6 main strategy categories. Likewise, the most frequently used strategy ("I use a bilingual English/Vietnamese dictionary", $\overline{X}$ = 4.94) goes to Determination category, which also ranks the first amongst the 6 main VLSs categories in the table of frequency ranking. According to Oxford's system of reference to mean score, its mean score of 4.94 shows that this strategy was "always or almost always used" by most of the students.

A close look at the table 4 reveals that amongst 9 most frequently used strategies, 6 strategies are used to consolidate a new word and 3 of them are used to discover the meaning of a new word. The 6 consolidation strategies: verbal repetition ($\overline{X}$= 4.3), study the spelling of a word ($\overline{X}$ = 4.26), written repetition ($\overline{X}$= 4.25), study the sound of a word ($\overline{X}$= 4.23), say the word aloud when studying ($\overline{X}$= 3.92) and continue to study the new English word overtime ($\overline{X}$= 3.74) occupy from the 2nd to the 7th positions in the ranking table of the top frequently used VLSs. The 3 VLSs to discover the meaning of a word, namely "use a bilingual English/Vietnamese dictionary", "ask the teacher for the Vietnamese translation of the word" and "ask classmates for the Vietnamese meaning of the word" rank the 1st, 8th and 9th amongst the most frequently used VLSs with the mean scores of 4.94, 3.68 and 3.58 respectively.

5.4. The least frequently used Vocabulary Learning Strategies

The 6 least frequently used VLSs amongst 58 strategy items are presented in table 5 as follows:

Table 5. *The least frequently used learning strategies*

Ranking	Strategy categories	Item No.	Vocabulary learning strategies	Mean score
1st	MEM	25	I use the Peg method.	1.82
2nd	COG	48	I use flashcards.	1.83
3rd	MEM	24	I use 'scales' for gradable adjectives.	1.87
4th	MEM	23	I use semantic maps.	1.89
5th	MEM	26	I use the Loci method.	1.90
6th	MEM	37	I use the Keyword Method - connecting the English word with a Vietnamese word that sounds similar.	2.00

It is noticeable that five amongst the six least used VLSs belong to Memory category, which ranks the 5th in the Ranking table of mean scores for 6 strategy categories (table 3). One Cognitive strategy which ranks the 2nd in this table is the strategy "use flashcards" with the mean score of 1.83. "Peg method", whose mean score is 1.82, is the least frequently used VLS. The other four strategies are "use scales for gradable adjectives" ($\overline{X}$=1.87), "use semantic maps" ($\overline{X}$=1.89), "Loci method" ($\overline{X}$=1.90) and "Keyword Method" ($\overline{X}$ = 2.00).

In brief, the findings about frequency of VLS use can be summarized as follows:

(1) In terms of the frequency of use of overall VLSs, 140 students who were investigated reported employing VLSs with medium frequency.

(2) Regarding the frequency of use of VLSs in the six main categories (DET, SOC, SOC, MEM, COG and MET), students reported using strategies at medium frequency level of use in each of these categories.

(3) However, students reported using each individual strategy item at different level of frequency: 33 items at medium frequency, 16 strategies at low frequency and 9 strategies at high frequency level.

(4) The most frequently used VLSs were: using a bilingual English/Vietnamese dictionary, using verbal repetition, studying the spelling of a word, using written repetition,

studying the sound of a word, say the word aloud when studying, continue to study the new English word overtime, asking the teacher for the Vietnamese translation of the word and ask my classmates for the Vietnamese meaning of the word.

(5) The least frequently used VLSs were: Peg method, using flashcards, using 'scales' for gradable adjectives, using semantic maps, Loci method and Keyword Method.

6. Implications and suggestions

Based on findings of the research, the writer has provided some recommendations with the hope of improving the English vocabulary teaching and learning at HDU, as follows:

Firstly, it is necessary that teachers be aware of their students' VLSs preferences and understand why they value certain kinds of strategies. The information about students' VLSs use and preference may be very helpful for teachers. Based on this awareness, teachers can design their lesson plans with the most suitable teaching techniques and methods which suit the needs and learning styles of their learners.

Secondly, teachers need to help students be aware of VLSs and teach them how to use VLSs effectively. Brown [1] states that students will become more successful and responsible for their own learning if they are taught how to use a strategy effectively and develop their own strategies which suit their learning purposes. In class discussions about VLSs, after students have listed their own techniques, teachers can write these VLSs on the board and help students identify the strategies. Asking students to complete the questionnaires about VLSs is also another effective way to raise students' awareness about VLSs. This can give them a wider selection of strategies to apply in learning new vocabulary.

Thirdly, students should be taught about how to use VLSs appropriately instead of learning vocabulary in whatever way they want. Teachers can spend one or two first lessons at the beginning of the English course to introduce and guide students how to use VLSs or they can integrate strategy training into the lessons during the course with different kinds of exercises.

In order to guide students how to use VLSs effectively, teachers can follow different strategy training models proposed by famous researchers in the language field [3] [11] [13] [14] [16]. Among them, Grenfell and Harris's strategy instructional model may be the most comprehensive one. Grenfell and Harris's model can be summarized into 6 steps as follows:

"(1) Awareness raising: The students complete a task, and then identify the strategies they used.

(2) Modeling: The teacher models, discusses the value of new strategy, makes checklist of strategies for later use.

(3) General practice: The students practice new strategies with different tasks.

(4) Action planning: The students set goals and choose strategies to attain those goals.

(5) Focused practice: The students carry out action plan using selected strategies; the teacher fades prompts so that students use strategies automatically.

(6) Evaluation: The teacher and students evaluate success of action plan; set new goals; cycle begins again."

(Grenfell and Harris, 1999; as cited in Jing Liu, 2010)

Lastly, teachers should design a variety of particular vocabulary exercises and activities to help students practise applying VLSs in class. Giving students more exercises to practice will help them be familiar with using new strategies. Then, with time, their use of VLS can become unconscious and automatic. As a result, students' vocabulary learning can be significantly improved.

7. Conclusion

This article aims at exploring the strategies used by non-English majored students at HDU and the frequency of VLS use of those learning strategies. The results from data analysis reveal that the most frequently used VLSs were: using a bilingual English/Vietnamese dictionary, using verbal repetition, study the spelling of a word, use written repetition, studying the sound of a word, say the word aloud when studying, continuing to study the new English word overtime, asking the teacher for the Vietnamese translation of the word and ask my classmates for the Vietnamese meaning of the word. The least frequently used VLSs were: Peg method, using flashcards, using 'scales' for gradable adjectives, using semantic maps, Loci method and Keyword Method.

Based on the findings, the researcher has provided some recommendations with the hope of improving the English vocabulary teaching and learning at HDU. By studying VLSs used by non-English majored students, this article provides English teachers at HDU with better background about their students' vocabulary learning. Teachers can find out the ways their students learn English vocabulary and which strategies they consider as the most useful. Based on this background comprehension, English teachers at HDU can create a more effective plan and adopt more suitable methods to teach their students.

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SIMULATION OF NANOPOROUS LOW-DENSITY STRUCTURES FROM $ZN_{12}S_{12}$ CLUSTERS

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Abstract: *Solid inorganic structures with the same composition but different polymorphisms often have different properties and applications, therefore, synthesizing and predicting new classes of polymorphisms for a given compound are of great significance and have been received considerable attention. In this paper, we propose a bottom-up approach to design nanoporous low-density structures from $Zn_{12}S_{12}$ clusters. Depending on the arrangement of the $Zn_{12}S_{12}$ clusters, the different polymorphic phases will be obtained. Our results show that the simulated structures are stable, do not collapse in the crystal, and are all semiconductors with wide band gap.*

Keywords: *Density functional theory, porous nanostructures, semiconductor materials.*

I. Introduction

Zinc sulfide (ZnS) is an II-VI semiconductor material, with a band gap of about 3.7eV. It has the advantages of having a relatively large band gap, the ability to withstand strong electric fields, high breakdown voltage, low electronic interference, and can operate at high temperatures with high power. ZnS nanomaterials have many physical and chemical properties which are not found in bulk semiconductors, such as band gap depending on the particle size, stable chemical properties and more applications in engineering than other materials [1]. In particular, when a transition metal ion such as Eu^{2+}, Cu^{2+}, Mn^{2+}, Co^{2+}... is doped, they can affect the structure and electronic level transition. Thus, it is possible to control the wide band gap, as well as to control other emission bands in the visible region of ZnS crystals when the impurity concentration and fabrication conditions of the sample are different. Therefore, this type of material has a wide range of applications, for example, in photoelectric devices, phosphor screens, optical sensors, etc.[2]. In its synthetic form, zinc sulfide can be transparent, and can be used as transparent windows to visible light or infrared glass [3].

Since discovery of amazing properties and many applications related to energy and sustainable development, such as catalysis, gas separation, water purification and fuel cells, the field of nanoporous materials has become one of the leading research directions. The main purpose of this research direction is to find materials that possess channels and features which make them become microporous or nanoporous. The most commonly used approach is the bottom-up approach, which starts from a cluster of molecules, by different

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combinations to reach the target structure [4] [5]. In many previous researches [6] [7], we have theoretically simulated several new nanoporous structures of ZnO materials by using the smallest nanoclusters as the basic elements to build up block structures.

Up to now, synthesizing a specific structure has been very important and in most cases remains a challenge. In this article, continuing the bottom-up approach, we will compute simulations based on density functional theory to build up some of nanoporous structures starting from the $Zn_{12}S_{12}$ molecular cluster. Then, we will also discuss the characteristic properties of those nanoporous structures.

2. Research Methods

2.1. Structural design method

In the framework of this paper, there are four polymorphic phases that we conduct to simulate and calculate to study their characteristic properties, denoted as SOD , FAU, LTA, AST[2]. The mentioned nanoporous structures all initiate from the $Zn_{12}S_{12}$ atomic cluster (Figure 1), which consists of atoms with the coordination number is three (different from the case in crystals is four), forms rings of even numbers of atoms - due to the equal roles of Zn and S in tetrahedral symmetry, that is, each Zn(S) atom is connected to three S(Zn) atoms by an ionic bonding. In particular, this cluster of $Zn_{12}S_{12}$ atoms consists of eight six-member rings (so called as 6-MR) and six four-member rings (so called as 4-MR), which obey the isolated tetrahedra rules which means there are two different types of Zn-S bonds in the atomic cluster: shorter in two adjacent ring bonds (4-MR) and longer inside the rings (4-MR).

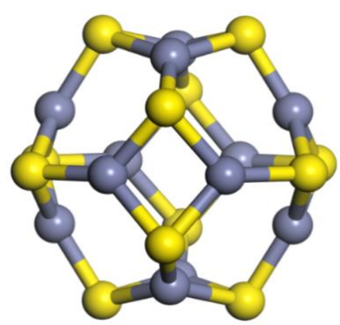

Figure 1. *The relaxed structure of $Zn_{12}S_{12}$ atomic cluster. The yellow balls are S atoms, the grey ones are Zn atoms*

The first SOD polymorphism phase (Figure 2) is created by linking $Zn_{12}S_{12}$ clusters through fourteen tight bonds between all rings of (6-MR) and (4-MR) (We define a tight binding as a linkage that shares a ring bond). When forming the crystalline phase, the number of coordinations of the atoms is four, so that there is no inward contraction since its covalent bonds have been saturated in all directions.

[2] This acronyms are zeolite framework type code according to the nomenclature of the International Zeolite Association (IZA)

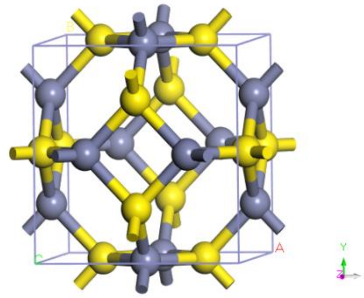

__Figure 2.__ The SOD polymorphic phase in crystal lattice primitive cell

Next, the FAU polymorphic phase (Figure 3) is also formed by binding the $Zn_{12}S_{12}$ clusters but differs in the binding method, specifically it consists of four double bonds of the (6-MR). (We define: A double bind is a bond through two bonding rings where the position of atoms do not overlap, close to each other to form bond bridges). Thus, it can be seen that if compared with the SOD polymorphic phase, the linkage between the $Zn_{12}S_{12}$ clusters in the FAU are fewer, less tight (because through the bond bridge rather than tight bond), thanks to this we can see more clearly in the specific bonding energy graph in the detailed calculation (Figure 6).

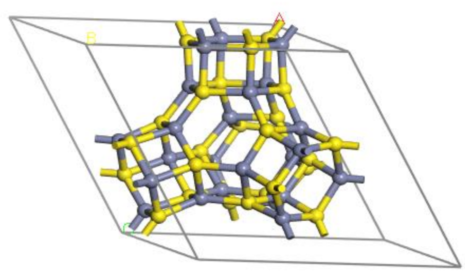

__Figure 3.__ The FAU polymorphic phase in crystal lattice primitive cell

For the LTA polymorphic phase (Figure 4), which is created through six double bonds of the (4-MR) in the $Zn_{12}S_{12}$ atomic cluster with six surrounding $Zn_{12}S_{12}$ atomic clusters. Since the bonding behavior between the $Zn_{12}S_{12}$ atomic clusters is different for each crystal phase, the lattice constants of the porous crystalline phases are completely different (Table 1).

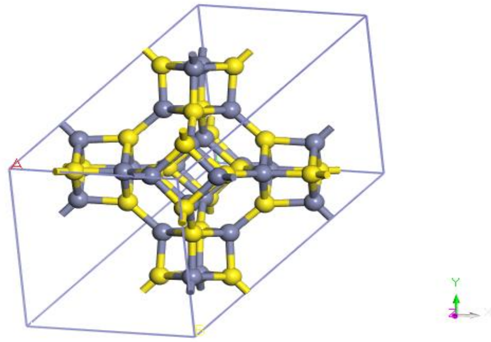

__Figure 4.__ The LTA polymorphic phase in crystal lattice primitive cell

For the final AST polymorphic phase (Figure 5), this structure is accompanied by six double bonds of the (4-MR) and eight tight bonds of the (6-MR), distinct from the SOD polymorphic phase with 14 bonds which are all tight bindings even though they come from the same structure. Therefore, the number of atoms in the unit cell is larger (40 versus 12), in addition, the unit cell size (lattice constant), band gap width, specific area of the pore, and the pore volume are also different (Table 1). This shows that SOD and AST are completely different polymorphic phases, although both are derived from the $Zn_{12}S_{12}$ structure, the only difference is in the method of bonding the tetragonal faces.

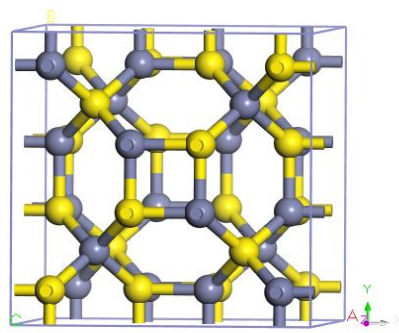

Figure 5. *The LTA polymorphic phase in crystal lattice primitive cell*

In summary, just from the same basic elements cluster, using the bottom-up approach, with varying the way of association we get different polymorphic phases.

2.2. Results and Discussions

Our calculation results are based on the Density Functional based Tight Binding (DFTB+) method [8, 9] associated with spin polarization and self-consistent charge. This method is based on the expansion of the quadratic approximation of the total energy functional in the Kohn-Sham electronic system depending on the spin of a given reference electronic system and the magnetization density.

2.2.1. Stability of the structure

Figure 6 shows that the most stable structure is the SOD structure, followed by AST, LTA and finally EMT. This result can be fully explained. Because of all the above structures, SOD is the structure with the most and tightest bonds between $Zn_{12}S_{12}$ clusters (based on 14 tight bindings in all rings), next the AST structure which also has the linkage between $Zn_{12}S_{12}$ clusters through 14 bonds but has both tight and double bonds, the remaining LTA and EMT structures are made up of $Zn_{12}S_{12}$ clusters but only by double bonds.

On the basis of these nonlinear cubic-fitted curves, we can draw some conclusions about the stability of the nanoporous structures. As we had expected, our proposed nanostructures are stable in all cyclic structural systems without collapsing (i.e. breaking) structurally which would lead to existence of their low density nano porous phases.

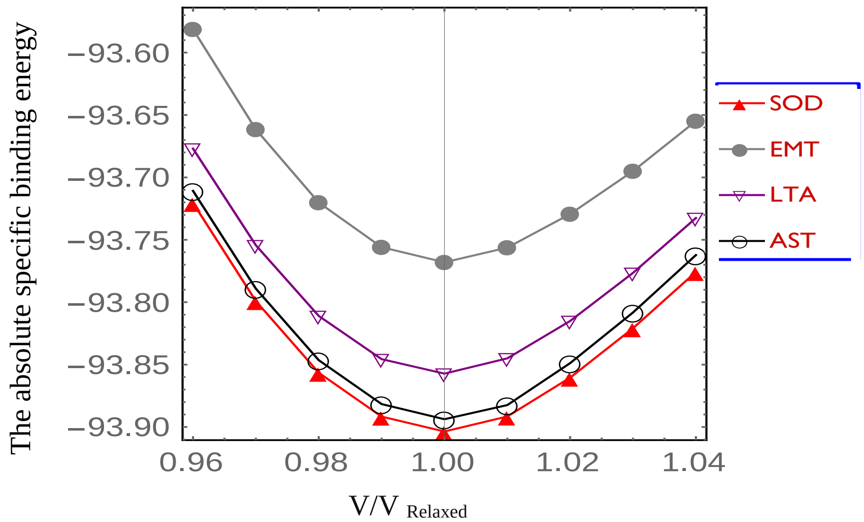

Figure 6. *The dependence of the absolute specific binding energy by the comparative volume*

Figure 7 shows asymmetric differences between the two parabolic branches of the curve in all the structures we have designed. In particular, it is clear that the left branch ($V/V_{relaxed} < 1$) is higher than the right branch, which qualitatively means that stretching will be easier (in terms of energy) than generating compression pressure in all investigated structures.

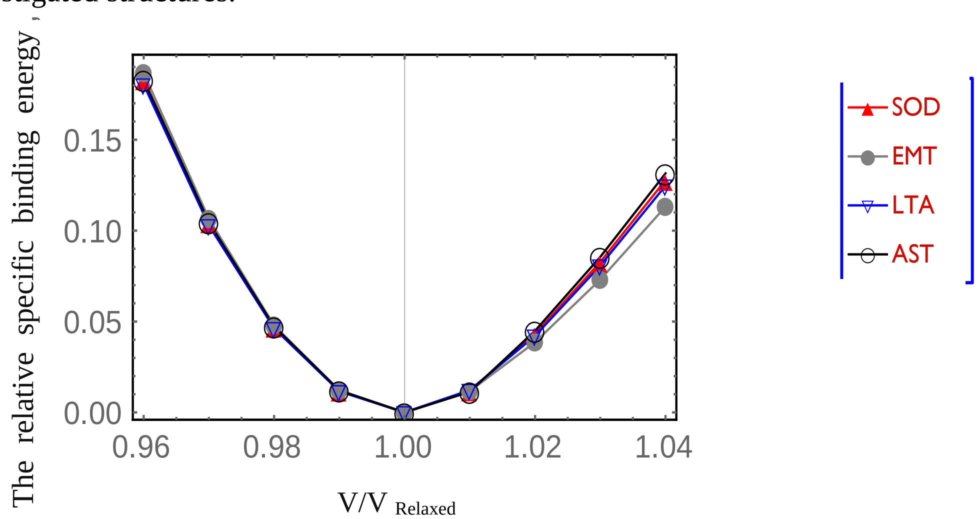

Figure 7. *The dependence of the relative specific binding energy by the comparative volume*

2.2.2. Energy band structure

In this section we study the effect of polymorphism on the electronic energy band structure. It can be seen from the figure 8, each different polymorphic phase has different bandgap and energy band structure.

With the parameterization of the tight binding approximation used in our calculations, the DFTB+ calculation obtains the region widths of the polymorph phases here ranging from 2.9eV to 3.2eV, the largest being the phase LTA polymorphism, and the smallest phase is EMT polymorphism. These wide band gap values are not much different from the band gap of ZnS materials (3.4eV) [11], that is, these polymorphic phases still retain the important and useful properties of bulk ZnS materials.

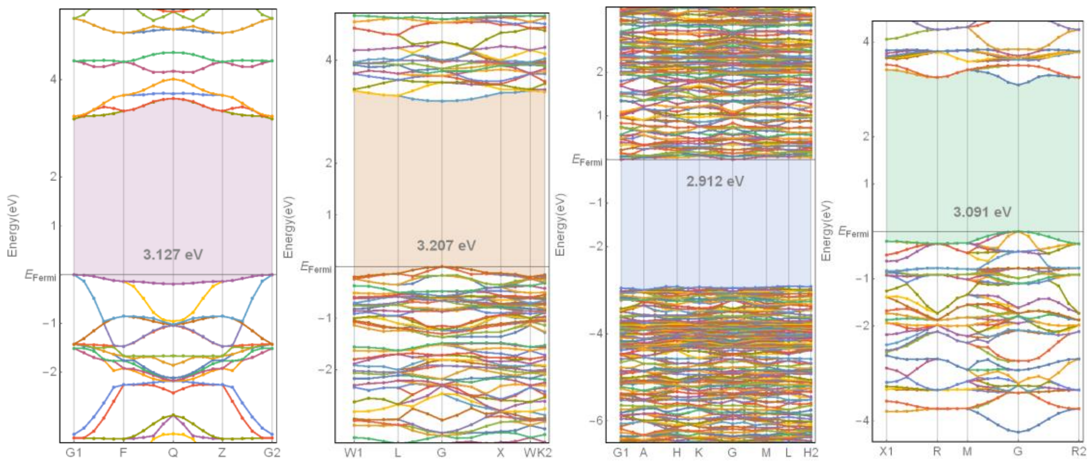

Figure 8. *From left to right is the band structure of SOD, LTA, EMT, AST phases*

2.2.3. Structural parameters

Table 1. *Summary table of structural parameters of ZnS nanoporous phases*

Structure	SOD	FAU	LTA	AST
Mass density (gcm^{-3})	4.48	3.18	3.65	4.19
Volume /at (Å^3/at)	14.38	20.21	17.82	15.53
Particle density ($10^{23}cm^{-3}$)	12.56	48.24	48.42	40.43
Coordination number	4	4	4	4
Crystal structure	Cub	Cub	FCC	Cub
Unit cell (atoms)	12	48	48	40
Lattice constant (*a*-*c*) (Å)	6.70	13.36	12.82	10.20
Average bond (Å)	2.027	2.040	2.032	2.031
Average angle Zn-S-Zn	120	110.4	121.09	125.62
Average angle S-Zn-S	120	110.2	118.82	124.91
Band gap (eV)	3.127	3.000	3.207	3.091
Surface area (Å^2)	120.03	533.57	546.31	407.69
Pore volume (Å^3)	94.44	865.97	674.0	351.2

As the results show in table 1, in all polymorphic phases-infinite three-dimensional cyclic crystal structures assembled from a series of independent $Zn_{12}S_{12}$ clusters having coordination numbers for all phases are quaternary, that is, each S(Zn) atom has four closest Zn(S) atoms to form a modified hybrid sp3 bond in almost all phases. That is, these polymorphic phases still retain important properties of ZnS materials such as: being a wide band gap semiconductor, piezoelectric, and having optical transparency to visible light. Although produced by the same clusters of $Zn_{12}S_{12}$ molecules, the results in table 1 show that each structure depending on how the different $Zn_{12}S_{12}$ clusters are linked gives us structures with completely different characteristics. Evidence for this is that the mass density, specific volume, particle density, lattice constant, mean bond length, mean bond angle, band gap width of porous structures are completely different. Naturally, each of these porous structures has completely different optical and mechanical properties. Here we also conduct a detailed assessment of the properties of hollow cages or porous walls through quantities such as the pore volume as well as the specific surface area of the energy-optimized or restored pore lattices of the unit cell or base cell of phases, the results allow an assessment of the porosity level of these polymorph phases to a certain extent. The results in table 1 show that in the designed structures, the SOD structure is the least porous and the EMT structure is the most porous. In addition, the results of the graphs in the figure 6 also indicate that the desgned porous nanostructures can exist sustainably.

3. Conclusion

Our study shows that the simulated polymorphic phases are stable, not collapsing in the crystal. It can be believed that the nanoporous phases produced by assembling clusters of atoms will become an attractive target for the synthesis of nanoporous polymorphic phases. The same comes from the $Zn_{12}S_{12}$ molecular cluster, but in different binding methods, it will give the nanoporous polymorphic phases with different band gap, surface area, pore volume, etc. This is an important factor in the experimental orientation as well as the practical application of porous nanostructures.

As a result, these new nanoporous phases, if synthesized, will be one of the most promising candidates to replace expensive, mechanically fragile or brittle materials. These porous structures can be used in molecular filtration, many applications in the field of catalysis, purification, oil refining technology etc. Due to their porosity and hollow cage structure, these structures can contain, purge, i.e. pass through or prevent and/or retain within its lumen atoms/molecules corresponding in size to pores or styroform.

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USING VIDEO RECORDING PROJECT TO IMPROVE PRONUNCIATION FOR NON-ENGLISH MAJOR STUDENTS AT A UNIVERSITY IN VIETNAM

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Abstract: *This article is about an action research which has been conducted in the course English 3 for 40 second-year students of Early Childhood Education Faculty, Hong Duc University. The research implemented the activity of self-recording as a homework after teacher instructed pronunciation rules and assessment criteria. The components of pronunciation which were focused are sounds, stress and intonation. There are two circles in this action research. After the first circle, there were supplemental actions to improve the intonation practice. The findings from tests, observation, and students' document express that the self-recording has had a positive impact on students' pronunciation when they pronounce English.*

Keywords: *Video recording, pronunciation, non-English major.*

1. Introduction

Pronunciation has long been considered as a very important part in the language learning process by many well-known scholars in the world. One among these scholars, Tench [6] already states that pronunciation is not an optional extra for the language learner any more than grammar, vocabulary or any other aspect of language is. In order to get the reasonable pronunciation of a language, firstly the speakers must have the acceptable understanding of the very basic foundation: the sound system, the syllable system and the word systems. After that, it is suitable to mention the idea about what factors can affect the acquisition of these systems in a certain second language. Avery and Ehrlich [1], on the other hand, claim that there are many factors, and among these, the teacher plays a very decisive role. They could positively affect students by showing them the clear purposes and realistic goals.

The students at Hong Duc University are required to have general knowledge in four main skills of English: reading, speaking, listening and writing. Although they do not have the main course book of pronunciation, this sub-skill does clearly affect students' speaking and listening results. To achieve the required level, the students obviously have to spend much time self-studying outside classroom. Besides, most of them were full of energy with pronunciation practice, and pleased to use the technology in class.

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2. Theoretical background

2.1. Pronunciation

2.1.1. Pronunciation concepts

Tench [6] defines pronunciation as a notion which is more than a matter of consonants, vowels and diphthongs. He links pronunciation to the sound of people when they are speaking, to the rise and fall of the pitch of the voice, to the pausing and grouping together of words and phrases, and to the highlighting of some syllables and the virtual suppression of others. Pronunciation, then, is itself a complex of sounds (consonants, vowels and diphthongs), syllables (word accent and rhythm) and intonation.

From a different view point, according to Hewings [3], there are certain components of speech which together combine to form the pronunciation of a language. These components range from the individual sounds that make up speech, to the way in which pitch - the rise and fall of the voice - is used to convey meaning.

2.1.2. Aspects of pronunciation

The following diagram of Kelly [5] shows a breakdown of the aspects of pronunciation:

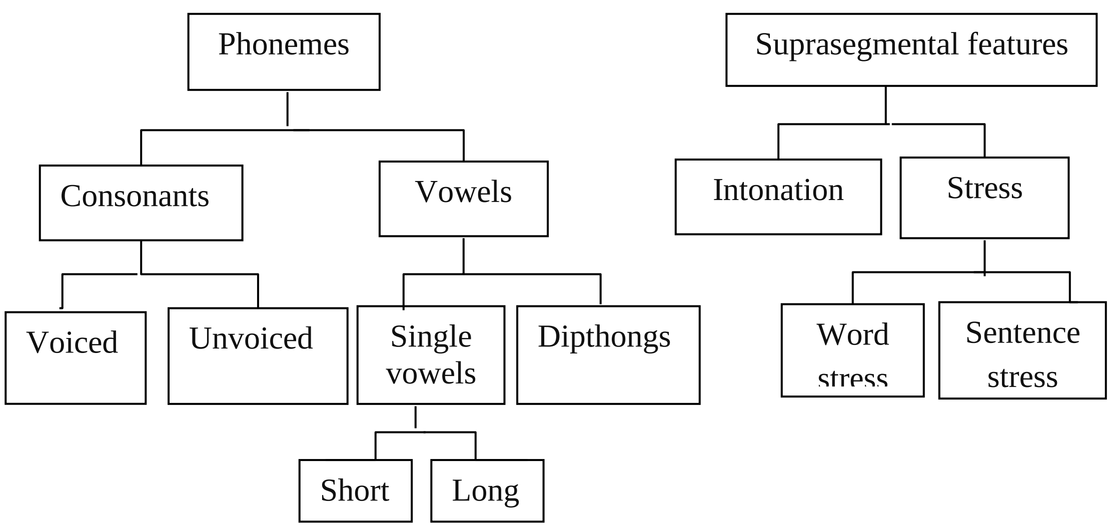

Figure 1. *Feature of pronunciation (Kelly, 2000)*

It is clear that Kelly sees pronunciation as the combination of two main parts: phonemes and suprasegmental features. The former component includes consonants and vowels, whereas the later one contains intonation and stress.

2.2. Teaching pronunciation

2.2.1. Teaching pronunciation

Different scholars suggest different approaches to teaching pronunciation. In the ways of teaching English pronunciation, there are some clear approaches. Tench [6] for example, suggests that the basic strategy in pronunciation teaching is imitation. The learners need to be able to imitate the teacher's pronunciation, imitating not only the features of consonants, vowels and diphthongs, but also rhythm and intonation.

Kenworthy [2] points out several factors which may affect pronunciation accuracy. They are native language, age factor, amount of exposure, phonetic ability, attitude and identity, motivation and concern for good pronunciation. He also tries to find out which factors can themselves be affected by teaching and training. Among them, motivation and concern for good pronunciation can be greatly affected.

2.2.2. Teaching pronunciation to Vietnamese learners

Hewings [3] acknowledges that in some classes, pronunciation is given a lower priority than other components of language such as grammar and vocabulary, and is sometimes relegated to an 'end-of-the-day' activity or a five-minute filler to give students some relief from 'real' work of language learning. This situation is also very common in many English lessons in Vietnam.

For most students, however, an understandable pronunciation will be an important part of their communication skills, and this justifies giving pronunciation a more central role in teaching by integrating it with other areas of language work. Perhaps, the most obvious area for useful integration is work that connects vocabulary and pronunciation. There are good arguments for teaching the pronunciation of words (both the sounds and their stress) as they are introduced. If students have confidence that they can pronounce a word correctly, they are more likely to use it as they speak, and using words successfully aids memorization [6].

2.2.3. Self-evaluation and monitoring

According to Kenworthy [2], teacher should combine the monitoring and self-evaluation both inside and outside classroom. Moreover, oral homework should be included.

Language learning is an extended process, and whenever we are involved in a learning task over a long period of time it is very easy to lose track of our progress. Therefore, keeping a record of progress or the students' tape will help.

Video recording as homework is mentioned as following:

Assignments are prepared, rehearsed, and then recorded on video or audio tape... learners evaluate their own performance using a score system specified by the teacher. There is then an opportunity for feedback and evaluation by the teacher. This may be given during 'pronunciation clinics' (ten or fifteen minute-slots when students can discuss their assignments) or during a lesson in which class members have the opportunity to play or show their completed assignments and receive comments from their teacher and colleagues.

In general, the theories of teaching English pronunciation focus on encouraging students to imitate the certain model and self-evaluating their practicing results gradually. Based on these theories, I constructed my research with the main principle of imitating and self-evaluating to see the changes. As the previous part, building the general understanding about sound, word and stress system would be the first step, then teacher should let student to imitate the model and evaluate their practice's product.

3. Research methodology

3.1. Research question

The study was carried out with the purpose of finding out the answers to the following research questions:

How do teachers implement video recording to improve current pronunciation practice of students?

How effective are these activities in improving the students' pronunciation and listening skill?

3.2. Participants

The study was carried out at Hong Duc University in Thanh Hoa Province. Hong Duc University is a public institution which includes many different faculties. The participants of the study are in one class of about 40 non-English major students in the department of Preschool Education, who had to study *English 3* as a compulsory subject in the academic year of 2019 - 2020 at Hong Duc University. The study was conducted for the purpose of implementing the intervention to improve the pronunciation by asking them to video-record themselves when pronouncing the English words or sentences at home as the assignment.

3.3. Data collection

3.3.1. Test recording

All the scores of tests of students were recorded. In every pronunciation test, students were required to recognize the separate sounds, words and sentences. The result of the test was noted by the teacher into the blog. The score of listening tests and pronunciation tests were kept in the teacher blog as well.

3.3.2. Classroom observation

The teacher observed, took note about the changes of students' pronunciation and kept the notes in the teaching log.

3.3.3. Students' document

The researcher collected the assessment sheets of the students after every meeting to analyze them. The improvement via the rate in the assessment sheet and the comments were selected to analyze. The documents also included the videos of students uploaded in the class page.

The data which were collected from classroom observation and students' documents were analyzed by qualitative method and quantitative method, the results of formal tests were analyzed by quantitative method.

4. Finding and discussion

4.1. Finding from the pre-tests and the first recording

From the result, the researcher realized most of students had problems with the initial voiceless /t/, the middle voiceless consonant and middle /s/ or the ending sounds. Moreover, even simple words were often pronounced with wrong stress syllable.

The first time self-recording videos of students were also used to analyze the main problems of pronunciation and then to compare with other videos to see the changes in their English pronunciation practice after the process of doing research.

In general, students in this study were at a quite low level of starting point in listening skill, most students did not have ability to recognize English words. Moreover, they lack of the basic knowledge of pronunciation like sounds or words stress. The intervention was going to be carried out to improve the situation.

4.2. The first intervention

The first circle of intervention took place during the first 5 weeks of the course. Based on the class observation and the pre-test, the researcher introduced the pronunciation exercises and asked students to practice at home individually and record it. The researcher also provided time for students to peer review and gave feedback after each recording session. Finally, the researcher distributed the marking scheme for students to help them correct themselves according to the marking criteria.

In general, after this intervention, there was a slight increase in the medium score of listening test, however, the higher one tended to decrease. The results of pronunciation test were improved significantly, but the production of single components of pronunciation had different changes, both positively, and negatively. When the separate sounds and stress were produced more correctly than the test 1, the correctness of intonation produced almost decreased. This would require more intervention after the first process of 5 weeks.

4.3. The next circle of intervention

The next circle of intervention took place from week 6 to week 11 in the English 3 course. Based on the class observation, the pre-test and the second test, the researcher kept introducing the intonation exercises and asked students to practice at home individually and record it. The main goal of this circle is to help students correct the intonation themselves and confirm the general trend of listening score in relation with the improvement of pronunciation itself.

After the second intervention, there was the similar trend of increase in medium score in listening test. The score of pronunciation test also significantly went up, but the main goal of improving intonation patterns was not achieved, except for a clear improvement in the rising tune of Yes/No question. This would require more intervention and method to help students correct themselves.

4.4. Results of students in the pretest, 2nd test and 3rd test in comparison

Table 1. *Results of mini-pronunciation tests*

	Test 1	**Test 2**	**Test 3**
Number of students	40	40	40
MEAN	1.54	4.24	6.6
MEDIAN	2	4	6.5
MODE	2	6	7

It is clear that results of pronunciation test improved significantly after every stage. The average correct words (MEAN) in 3rd test was 6.6, which is approximately three times higher than that in the 1st test. The center score (MEDIAN) and the score which appeared with the highest density (MODE) also tripled after the intervention.

4.5. Further findings from teacher's observation and students' documents

The checklist of teacher in observation sheet can be sorted in the following table to compare the students' behavior before and after the intervention to see the degree of participation and pronunciation awareness. The behaviors are compared in the stage of before and after each intervention. The observations were taken in class, when the teacher called students to practice, or when teacher went around to see students working in pair or group.

Table 2. *Behaviors of students in two intervention cycles*

Student's behaviors	**Before intervention**	**After the 1st intervention**	**After the 2nd intervention**
Slowly standing up	90%	77.5%	65%
Not paying attention to instruction in pronunciation	27.5%	25%	25%
Slowly pronouncing	92.5%	47.5%	32.5%
Not raising hand when T calls volunteer	100%	92.5%	57.5%
Being careful in producing first consonant	2.5%	27.5%	40%
Being careful in producing final consonant	0%	22.5%	42.5%
Being careful in producing middle /s/	7.5%	22.5%	32.5%
Paying attention to word stress	10%	40%	37.5%
Paying attention to sentence stress	0%	15%	17.5%
Paying attention to intonation pattern	7.5%	47.5%	47.5%

There were the significant changes in the students' behaviors. The negative behaviors like slowly standing up or pronouncing slowly were decreased (from 90% to 65% and 92.5% to 32.5% respectively). Moreover, the positive behaviors like paying attention to sound /s/ or word stress increased (from 7.5% to 32.5% and 10% to 37.5% respectively).

Teacher's observation during class hour and informal interview with students in break time was considered as a supportive data to find out the effect of using video recording on student's pronunciation and the way they produce sounds, words and sentences. Besides, it shows how students evaluated the new method of self-recording and how helpful they thought it was. The findings are presented as follows.

Firstly, students were embarrassed because they had never recorded themselves and let others see before. Then they were encouraged by the promise of teacher that the Facebook group would be private and nobody else except for their classmates and teacher could see those videos. After that, students were very enthusiastic to join in and almost all the students in class participated in the recording group. At the beginning, a lot of sounds and words were pronounced wrongly when students had to spend much time before reading each word; in the end, they could produce some sounds very similar to those of English native speakers.

Secondly, during the class hour, students were really motivated. By giving and receiving feedback, they paid more attention to the components of pronunciation like sound, intonation and stress. The comments of teacher on sample video of a random student every period also helps the students repair the common mistakes. On the other hand, the informal interview with students reveals that they thought the video recording activity was very new and special. Even though, this activity helps them very effectively in motivate self-speaking.

For the intonation field, students often forgot to apply the knowledge about types of intonation when practicing the conversations. Even after the instruction, students seemed to be shy to use the correct intonation, especially the rising ones. The situation was improved when students had to practice and record in pairs. In this way, the confidence of learners was increased since they could have one more partner also appear and they could do as they were in real life.

Students also expressed the significant improvement on evaluation sheets after two periods of intervention.

Table 3. *Student's documents of self-evaluation sheets*

Criteria	Before intervention	After the 1st intervention	After the 2nd intervention
Pronouncing the separate sounds correctly	57.5%	65%	65%
Pronouncing the separate words correctly	15%	17.5%	25%
Pronouncing the separate sentences correctly	0 %	7.5%	5%
Having correct word stress	35%	77.5%	90%
Having correct sentence stress	27.5%	22.5%	52.5%
Having correct intonation	0%	15%	32.5%

It is clear that after 2 interventions, the percentage of correctness increased significantly, especially in pronouncing separate words (from 15% at the beginning to 25% in the end) and word stress (from 35% to 90%). The improvement was that the percent of correct sentence stress and intonation rose (from 27.5% to 52.5% and from 0% to 32.5% respectively).

5. Conclusion and implications to language teaching and learning

In this paper, an action research has been conducted in the course English 3 for 40 second-year students of Early Childhood Education Faculty, Hong Duc university. The components of pronunciation were focused are sounds, stress and intonation. Certain exercises including practicing sounds, words, short conversations and listing the words were introduced. The instruments of conducting the research are record of pretest, 2nd tests and 3rd test, class observation and students document with informal interviews. The researcher first investigated the situation of pronunciation and listening level of students through pretest, and some special features of pronunciation or common mistakes were also discovered when the first video of every learner was analyzed, mostly the mistake when producing the first consonant /t/, and middle /s/ or the final sound. There are 2 circles in the study. After the first 5 weeks, there was a supplemental actions of asking students to record in pair to improve the intonation practice. The findings from tests, observation, interviews and students' document have shown that the self-recording improves pronunciation and the confidence for students when pronouncing greatly.

This study can be used as a reference source for English teacher, students and those interested in using new technology for self-study. The findings of the study indicated that technology could play as a positive factor to motivate the English learning environment of students. Even though there is a new action, students will very quickly adapt with it. Video self-recording activity is very practical homework which can be applied to improve the confidence and pronunciation proficiency. However, it must go well with the careful instruction and research on the common mistakes. Teacher could let students evaluate their videos so that they could check their pronunciation again in class.

During the first time of recording, students, especially non-English major ones often feel embarrassed and do not dare to record. What the teacher should do is to encourage them, let a closed and secured page so that nobody outside the class could see these videos. In addition, there must be the bonus marks for those who have good videos.

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COMPARISON CHARACTERIZATIONS OF Cr (III) AND Cr (VI) CONVERSION COATING ON ZINC ELECTROPLATING SUBSTRATE

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Abstract: *Some of characterizations of Cr (III) conversion coating have been compared with that of Cr (VI) conversion coating. The comparison was made on the base of conversion coating color, anticorrosion ability, and effect of drying temperature. The obtained results indicated that the color of Cr (VI) conversion coating was darker than that of Cr (III) conversion coating. However, the color of Cr (VI) conversion, more or less, depended on the drying temperature. The color of conversion coating the darkest if the drying temperature was at 50^oC. Moreover, the characterizations of Cr (VI) depended significantly on the drying temperature. The drying temperature had a significant effect on anticorrosion ability of Cr (VI) conversion coating, especially for samples dried at higher temperature than 110^oC. In contrast, the Cr (III) conversion coating showed more stable and insignificantly depended on drying temperatures. In addition, the surface of Cr (III) conversion samples was smooth, no crystalline structure. Conversion coating was layered fibrous. The Cr (III) sample had no crash while the Cr (VI) had. Hence, the obtained results indicated that Cr (III) conversion solution can creat the conversion coating which has anticorrosion ability, which can replace Cr (VI) based conversion coatings.*

Keywords: *Cr (III) conversion, Cr (VI) conversion, coating, zinc electrodeposited, anticorrosion.*

1. Introduction

Zinc plating is widely used to go against corrosion for steel due to the low cost and simple technology. However, the corrosion rate of zinc coating might be very high in a humid environment since zinc is a highly chemical reactive metal [1]. Therefore, a post-treatment is necessary to increase the lifetime of zinc coatings. One of the most popular methods was to use Cr (VI) conversion solution to create a thin conversion coating on the surface of zinc plating because of many advantages, such as high anticorrosion, self-healing ability, adversity color (white, rainbow color, black…), good adhesion with organic coating, simply engineering and low cost... However, the compound Cr(VI) has been convinced as a hazardous substance that may cause cancer. The content of Cr (VI) in conversion layer fluctuates from 5 - 400 mg/m^2 in the using process, the Cr (VI) compound would be dissolved and cause of pollution [2] [3].

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Hence, many other treatment methods have been presented with requirements to replace Cr(VI) based conversion coatings with safer treatments in which Cr (III) conversion coating is introduced and become popular in industrial practice [4] [5].

With the purpose to build a system of research and testing the conversion coating based on Cr (III) conversion solution, in this work, we would present the results of the comparison of characterizations of Cr (III) and Cr (VI) conversion coating on a zinc electrodeposited substrate.

2. Experimental

2.1. Materials

The chemical materials used including: NaOH, HNO_3 (both used pure grade (China)), iridescent Cr(III) conversion complex solution (containing: Cr^{3+} in type $Cr_2(SO_4)_3.6H_2O$ at 5g/L, complexion agent at 16g/L and pH at 1.5), Cr (VI) conversion solution UDYCRO 747 was purchased from Enthone. Zinc electroplating was fabricated according to the ENTHONE process. The components of zinc plating solution are $ZnCl_2$ (60g/L); NH_4Cl (250g/L); additive AZA (30ml/L) and AZB (1.5ml/L).

2.2. Sample preparation

Steel low carbon plates (100x50x1.2 mm) were degreased by immersing in UDYPREP-110EC (Enthone) with concentration of 60 g/L under 50-60^{o}C for 5-10 min. After that the samples were immersed in a solution containing HCl (10%), urotropin (3.5 g/L) at ambient temperature for 2-5 min.

The steels were industrially electro galvanized in plating bath with a solution of Enthone Company. The conditions were followed: cathodic current density of 2 A/dm^2; the zinc anode with a purity of 99.995%; rate of square anode/cathode of 2/1 and at ambient temperature for 30 min with the swinging cathode operation. Subsequently, the samples were rinsed with deionized water. Zinc coating had thickness of 12-13 µm

Immediately after the electro galvanizing step, the sample surface was activated in a 0.5% HNO_3 solution (pH = 1) for 3-5 s. Subsequently, the surface was passivated by either the following treatments green-colored Cr^{3+} based conversion treatment with parameters being 2-2.5 of pH, in 60 s or Cr (VI) conversion solution with parameter being 1,4-1,8 of pH during 30s in industrial immersion bath with mechanical stirring. Finally, the samples were rinsed in deionized water and dried in an oven at various temperatures for 30 min. All samples were stored in desiccators at ambient temperature in 48 h for stabilized samples.

2.3. Analysis

The general appearance of conversion coating was assessed after 48h for stabilizing samples. The corrosion resistance was evaluated by natural salt spray testing according JIS H 8502:1999 standard, which means by Q-Fog CCT 600 (USA) at Institute for Tropical Technology.

Morphology of conversion coating was obtained by HITACHI S-4800 Scaning electron microscopy.

Corrosion behavior of passive coating was determined by polarization method which means on Autolab Pgstat 30 using 3 electrodes with parameters were: 4.52 cm^2 of square sample, a platinum counter electrode, saturated calomel electrode as reference electrode, 2mV/s of scan rate, 3,5% NaCl solution, in presence of air.

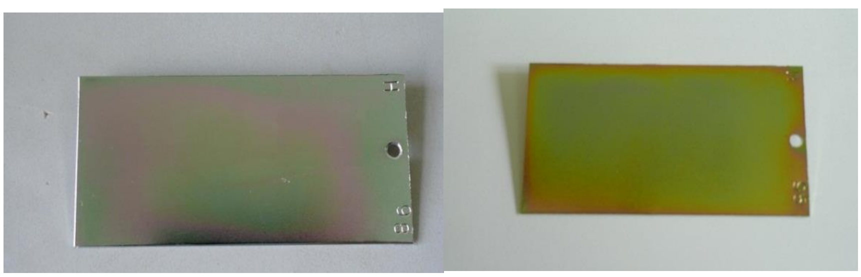
3. Results and discussions

3.1. The color of conversion coating

The appearances of conversion were obtained by either eyes and taking a photo. Table 1 presents the results of general appearance and color of conversion coating.

Table 1. *General appearance of conversion coatings*

From passive solution	***Notation***	***Appearance***
Iridescent Cr (III) conversion complex solution	C	Glossy surface, having the light rainbow color with main the color being pink and light green.
Cr (VI) conversion solution UDYCRO 747	747	Glossy surface, having the dark rainbow color with the main color being light blue and golden.



(C) (747)

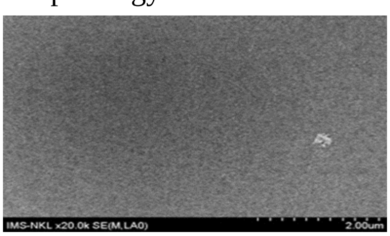
Figure 1. *The color of conversion coatings*

As can be seen from Table 1 and Fig.1: There was a significant difference in color between conversion coating from C conversion and 747 solution.

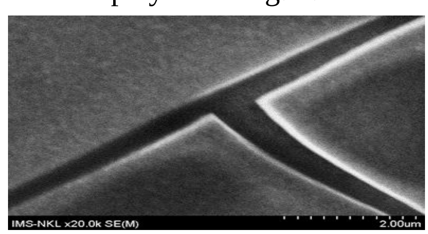
The color of conversion coatings from Cr (VI) conversion solution was darker than that of conversion coating from Cr (III) conversion solution.

3.2.Morphorlogy of conversion coating

Morphorlogy of conversion coating surfaces was displayed on Fig. 2.



a) C



b) 747

Figure 2. *Mophorlogy of Cr(III) and Cr(VI)conversion coating surfaces*

From Fig. 2. It can be seen that both of conversion coating surfaces are smooth and do not show crystal.

For the Cr (III) conversion coating, the surface has no crash. In contrast, the surface of Cr (VI) conversion coating had some crash with about 200 nm in diameter. It may be due to the pull-push of conversion coating if the surface was dehydrated by drying process.

3.3. Effect of drying temperature and anticorrosion of conversion coating

3.3.1. Weight loss

The weight change of conversion coating in various temperatures of the drying process were shown in table 2.

Table 2. *The weight change of conversion coating in various temperatures*

Temp / **Conversion coating**	**Weight loss (%)**			
	50^oC	80^oC	110^oC	210^oC
747	9.7	14.3	23.6	28.4
C	8.6	11.4	17.1	24.3

As can be seen from table 2, the weight loss of conversion coating increased if the drying process temperature rises from 50 - 210^oC. The weight of conversion coating dramatically reduced in the temperature range from 50 - 110^oC. After that, it decreased slightly in the temperature range from 110 - 210^oC in comparison with the previous period. The weight change is the cause of dehydration from conversion coating when the samples were dried. As a result, if the temperature was higher than 110^oC, the amount of water in the conversion coating was insignificant.

3.3.2. Natural salt spray testing

The average time of white rust appear of conversion coatings were indicated on table 3. As can be seen from table 3: The highest anticorrosion of conversion coating depended on temperature in the drying process, at 80^oC for Cr (III) and 50^oC for Cr (VI).

If the drying temperature was higher than 110^oC, the corrosion resistance substantially reduced. A standard example was Cr (VI) conversion coating. After the sample was dried at 210^oC, the white rust appeared after 24h natural salt spray.

Table 3. *The time of white rust appear*

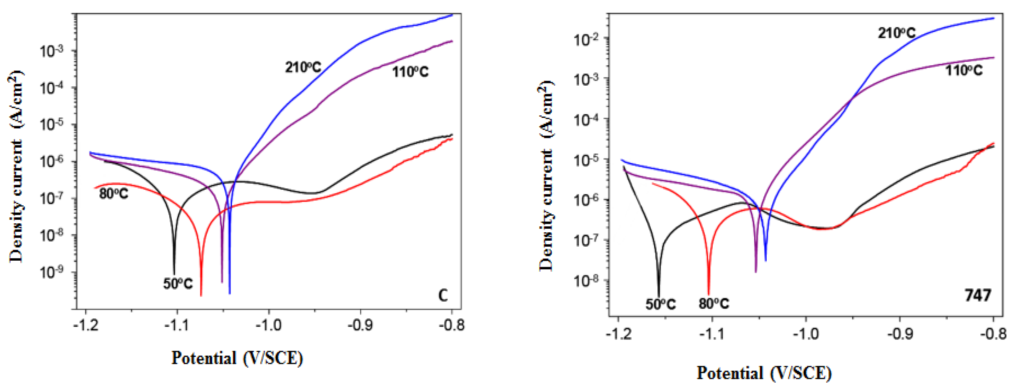
Drying temp. / **Coating**	**The time of white rust appear on surface, hour**			
	50^oC	80^oC	110^oC	210^oC
C	194	218	195	175
747	215	168	72	24

The obtained results indicated that the drying temperature had less effect on the Cr (III) conversion coating in comparison with on Cr (VI) conversion coating. The degradation of Cr (VI) conversion coating when increasing drying temperature might be the cause of Cr (VI) change in conversion coating. If the drying temperature was higher than 50^{o}C, the content of Cr (VI) dissolving might be changed or Cr (VI) could be converted to Cr (III) and thus reducing the anticorrosion ability.In the natural salt spraying process, it was obtained that the color of sample which dried at 50^{o}C became lighter than initial sample. It is obvious that the amount of Cr (VI) in conversion coating, to some extent, dissolved. In contrast, the sample dried at higher temperatures, the color saw insignificant change during the natural salt spraying process.

3.3.3. The results of polarization measurement

The curves of dynamic polarization of conversion coating samples from Cr (III) and Cr (VI) conversion solutions would be illustrated in Fig. 3 and Fig. 4. The potential value, corrosion density current were shown in table 4.

For the conversion coating samples from Cr (III) conversion solution, current density reduced as follow arrange of drying temperature: 210^{o}C; 110^{o}C; 50^{o}C; 80^{o}C. These results also were suitable with the obtained results in table 3. The anticorrosion can be arranged as 50^{o}C ~ 80^{o}C > 110^{o}C > 210^{o}C.



a) b)

Figure 3. *Polarization curves of conversion coatings produced in a - Cr (III)and b - conversion solution with various drying temperatures*

Table 4. *E_{corr} and i_{corr} values of samples*

Conversion solution	Drying temp., ^{o}C	E_{corr}, mV/SCE	i_{corr}, A/cm^2
C	210	-1046	$5,87\times10^{-7}$
	110	-1053	$1,52\times10^{-7}$
	80	-1074	$3,35\times10^{-8}$
	50	-1104	$6,6\times10^{-8}$
747	210	-1044	$2,46\times10^{-6}$
	110	-1055	$3,19\times10^{-6}$
	80	-1074	$4,29\times10^{-7}$
	50	-1157	$1,94\times10^{-7}$

The current density values of two samples coatings by 747 solution and dried at 110 and 210^{o}C were the highest. These results were compatible with the obtained results of natural salt spraying test.

The samples coatings by 747 solution and dried at 50^{o}C having i_{corr} that is higher than that of sample coatings by C solution which was dried at 80^{o}C (table 3). These results were compatible with the obtained results of natural salt spraying test.

However, the natural salt spraying results showed that the anticorrosion of samples coatings by 747 solution and dried at 50^{o}C was better than that of the samples coatings by C solution and were dried at 50^{o}C. It can be explained that the polarization measurement was determined at the initial. Moreover, this measurement was applied in a small square which may have disabilities while the natural salt spraying test was applied to all samples and some disabilities, more or less, had not effect on results.

4. Conclusion

The color of Cr (VI) conversion coating was darker than that of Cr (III) conversion coating.

The surface of passivated samples was smooth, with no crystalline structure. Conversion coating was layered and fibrous. The Cr (III) sample had no crash while the Cr (VI) had.

The drying temperature had a significant effect on anticorrosion ability of Cr (VI) conversion coating, especially for samples dried at higher temperature than 110^{o}C. In contrast, the Cr (III) conversion coating showed more stable and insignificantly depended on drying temperatures.

The obtained results indicated that conversion Cr (III) solution fabricated can create the conversion coating which has anticorrosion ability as conversion from conversion Cr (VI) solution.

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GLOBAL ATTRACTOR FOR A NONLOCAL p-LAPLACE PARABOLIC EQUATION WITH NONLINEARITY OF ARBITRARY ORDER

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Abstract: *In this paper we consider a nonlocal p-Laplace parabolic equation depending on the L^p norm of the gradient with nonlinearity of arbitrary order. First, we prove the existence and uniqueness of weak solutions by combining the compactness and monotone methods and the weak convergence techniques in Orlicz spaces. Then, we prove the existence and regularity of a global attractor for the associated semigroup. The main novelty of our results is that no restriction on the upper growth of the nonlinearity is imposed.*

Keywords: *Nonlocal p-Laplace parabolic equation; nonlinearity of arbitrary order, weak solution, global attractor, compactness method, monotone method, weak convergence techniques.*

1. Introduction

Let Ω be a bounded domain in $\mathbb{R}^N$ with Lipschitz boundary $\partial\Omega$ and let $p \geq 2$ be fixed. We consider the following quasilinear parabolic equation with nonlocal diffusion term

$$\begin{cases} u_t - \operatorname{div}\left(a\left(\| \nabla u \|^p_{L^p(\Omega)}\right) |\nabla u|^{p-2} \nabla u\right) + f(u) = g(x), & x \in \Omega, t > 0, \\ u(x,t) = 0, & x \in \partial\Omega, t > 0, \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases} \tag{1.1}$$

where the diffusion coefficient a, the nonlinearity f, and the external force g satisfy the following conditions:

(H1) $a \in C(\mathbb{R}, \mathbb{R}_+)$ and there are two positive constants m and M such that

$$0 < m \leq a(s) \leq M, \quad \forall s \in \mathbb{R}. \tag{1.2}$$

Moreover, we assume that

$$s \mapsto a(s^p)s^{p-1} \text{ is nondecreasing.} \tag{1.3}$$

(H2) $f : \mathbb{R} \to \mathbb{R}$ is a continuously differentiable function satisfying

$$f(u)u \geq -\mu u^2 - c_1, \tag{1.4}$$

$$f'(u) \geq -\ell, \tag{1.5}$$

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where μ, c_1, ℓ are positive constants, and if $p=2$ then we assume furthermore that $0<\mu<m\lambda_1$ with $\lambda_1>0$ being the first eigenvalue of the Laplace operator $-\Delta$ in Ω associated with the homogeneous Dirichlet boundary condition.

(H3) $g \in L^2(\Omega)$.

Equation (1.1) is nonlocal due to the structure of the diffusion coefficient which depends upon the L^p -norm of the gradient. In the last decade, a lot of attention has been devoted to nonlocal parabolic problems. One of the justifications of such models lies in the fact that in reality the measurements are not made pointwise but through some local average. Some interesting features of nonlocal parabolic equations and systems and more motivation are described in [1] [6] [7] [8] [9] [22] [27] and references therein.

On the other hand, the existence and long-time behavior of solutions in terms of the existence of global attractors to quasilinear parabolic equations involving p -Laplacian type operators have been extensively studied in recent years. A typical example of nonlinearity is the one satisfying a growth and dissipative condition of polynomial type

$$c_1 |u|^p - c_0 \le f(u)u \le c_2 |u|^p + c_0,$$
$$f'(u) \ge -\ell,$$

for some $p \ge 2$, see e.g. [2, 3, 5, 10, 11, 14, 17, 24, 26]. We notice that this class of nonlinearities requires some restrictions on the upper growth, and in particular, the exponential nonlinearity, for example, $f(u)=e^u$, do not hold.

For nonlocal p -Laplace parabolic equations, in some recent works [8] [9], Caraballo et. al. considered the following equation

$$u_t - a(l(u))\operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right) + f(u) = g(x),$$

where $a: L^2(\Omega) \to \mathbb{R}$ is a continuous linear functional, $f(u)$ is sublinear or is growth and dissipative of polynomial type. They proved the existence of global attractors in both cases with and without uniqueness of solutions, and these results are in some sense as extensions of previous ones for the following nonlocal reaction-diffusion equation in [4] [6] [20] [22].

$$u_t - a(l(u))\Delta u + f(u) = g(x).$$

While in [12, 13], Chipot and Savitska considered the following parabolic equation

$$u_t - \operatorname{div}\left(a\left(\|\nabla u\|^p_{L^p(\Omega)}\right)|\nabla u|^{p-2}\nabla u\right) = g(x),$$

with zero Dirichlet boundary conditions, where $g \in W^{-1,q}(\Omega)$. They proved the existence, uniqueness and long-time behavior of solutions to this problem. This result was extended in [25] with the nonlinearity satisfying the dissipative condition of polynomial type.

In this paper, we extend the results in [4, 12, 13, 25] by adding a nonlinearity of arbitrary order. Here we are able to prove the existence and uniqueness of weak solutions and the existence of a global attractor for a very large class of nonlinearities that particular covers both sublinear and polynomial type classes and even exponential nonlinearities. The absence of the upper growth condition on f and the diffusion coefficient determined by a global quantity causes a number of difficulties which make the analysis of the problem interesting. To overcome these essential difficulties, we exploit the weak convergence techniques in Orlicz spaces [17], and combine it with the standard monotone and compactness methods. In particular, the results obtained here are extensions of many previous results in [16] [24] [26] for local p-Laplace parabolic equations.

The paper is organized as follows. In Section 2, we prove the existence and uniqueness of global weak solutions to problem (1.1). In Section 3, we show the existence of global attractors in various spaces for the continuous semigroup associated to problem (1.1).

2. Existence and uniqueness of weak solutions

Let us denote $\Omega_T := \Omega \times (0,T)$ and let (p,q) be conjugate, i.e., $\frac{1}{p}+\frac{1}{q}=1$. First, we give the definition of weak solutions to problem (1.1).

Definition 2.1. *Let* $u_0 \in L^2(\Omega)$ *be given. A function* u *is called a weak solution of problem (1.1) on the interval* $(0,T)$ *if* $u \in L^p(0,T;W_0^{1,p}(\Omega)) \cap C([0,T];L^2(\Omega))$, $\frac{du}{dt} \in L^q(0,T;W^{-1,q}(\Omega)) + L^1(\Omega_T),\ f(u) \in L^1(\Omega_T), u(0)=u_0$, *and*

$$\int_\Omega \left(u_t v + a(\| \nabla u \|_{L^p(\Omega)}^p) |\nabla u|^{p-2} \nabla u \cdot \nabla v + f(u)v - gv \right) dx = 0,$$

for all test functions $v \in W_0^{1,p}(\Omega) \cap L^\infty(\Omega)$ *and for a.e.* $t \in (0,T)$.

As in [12], under the assumption **(H1)**, one can check that the operator

$$Au := -\mathrm{div}\left(\mathrm{a}(\| \nabla \mathrm{u} \|_{\mathrm{L^p}(\Omega)}^{\mathrm{p}}) |\nabla \mathrm{u}|^{\mathrm{p-2}} \nabla \mathrm{u} \right)$$

is monotone in $W_0^{1,p}(\Omega)$, i.e., for all $u, v \in W_0^{1,p}(\Omega)$, we have

$$\langle Au - Av, u - v \rangle \geq 0. \tag{2.1}$$

In addition, for each $u \in W_0^{1,p}(\Omega)$, we have the following inequality

$$\lambda_1 \| u \|_{L^p(\Omega)}^p \leq \| u \|_{W_0^{1,p}(\Omega)}^p . \tag{2.2}$$

In the case $p > 2$, it follows from the embedding $L^p(\Omega) \subset L^2(\Omega)$ and the inequality (2.2) that

$$\| u \|_{L^2(\Omega)}^2 \leq |\Omega|^{\frac{p-2}{p}} \| u \|_{L^p(\Omega)}^2 \leq |\Omega|^{\frac{p-2}{p}} \lambda_1^{-\frac{2}{p}} \| u \|_{W_0^{1,p}(\Omega)}^2 .$$

As an application of the Young inequality with ε, we obtain

$$\| u \|_{L^2(\Omega)}^2 \leq \varepsilon \| u \|_{W_0^{1,p}(\Omega)}^p + \frac{(p-2)|\Omega|}{p} \left(\frac{p \lambda_1 \varepsilon}{2} \right)^{-\frac{2}{p-2}} . \tag{2.3}$$

Using the Holder inequality, inequality (2.2), the Young inequality and the embedding $L^p(\Omega) \subset L^2(\Omega)$, we have the following inequality

$$\left|\int_\Omega gudx\right| \le \varepsilon \| u \|^p_{W_0^{1,p}(\Omega)} + \frac{|\Omega|^{\frac{(p-2)q}{2p}}}{q(p\varepsilon\lambda_1)^{\frac{q}{p}}} \| g \|^q_{L^2(\Omega)}, \tag{2.4}$$

for all $u \in W_0^{1,p}(\Omega)$ and any $\varepsilon > 0$.

Theorem 2.1. *Under the assumptions **(H1)-(H3)**, problem (1.1) has a unique global weak solution u satisfying*

$$u \in C([0,\infty); L^2(\Omega)) \cap L^p_{loc}(0,\infty; W_0^{1,p}(\Omega)),$$
$$\frac{du}{dt} \in L^q_{loc}(0,\infty; W^{-1,q}(\Omega)) + L^1_{loc}(0,\infty; L^1(\Omega)).$$

Moreover, the mapping $u_0 \mapsto u(t)$ is continuous on $L^2(\Omega)$, that is, the solution depends continuously on the initial data.

Proof. **i) Existence.** Fix $T > 0$ arbitrarily. Let $\{e_j\}_{j=1}^\infty$ be a basis of $W_0^{1,p}(\Omega) \cap L^\infty(\Omega)$, which is orthornomal in $L^2(\Omega)$. We look for an approximate solution $u_n(t)$ of the form

$$u_n(t) = \sum_{j=1}^n \gamma_{nj}(t) e_j$$

that solves the following problem

$$\begin{cases} \int\limits_\Omega \left[u_{nt} e_k + a\left(\| \nabla u_n \|^p_{L^p(\Omega)} \right) |\nabla u_n|^{p-2} \nabla u_n \cdot \nabla e_k + f(u_n) e_k \right] dx = \int\limits_\Omega g e_k dx, \\ \sum\limits_{k=1}^n \gamma_{nk}(0) e_k \to u_0 \text{ in } \mathrm{L}^2(\Omega) \text{ as } \mathrm{n} \to \infty. \end{cases} \tag{2.5}$$

Since $a \in C(\mathbb{R}, \mathbb{R}_+)$ and $f \in C^1(\mathbb{R})$, the Peano theorem ensures the existence of approximate solutions $u_n(t)$ on an interval $[0, T_n) \subset [0, T]$.

We now establish some *a priori* estimates for u_n. Multiplying the first equation in (2.5) by $\gamma_{nj}(t)$ and summing from $j = 1$ to n, we obtain

$$\frac{1}{2}\frac{d}{dt} \| u_n \|^2_{L^2(\Omega)} + a\left(\| \nabla u_n \|^p_{L^p(\Omega)} \right) \int_\Omega |\nabla u_n|^p \, dx + \int_\Omega f(u_n) u_n dx = \int_\Omega g u_n dx. \tag{2.6}$$

Hence, by (1.2) and (1.4), we have

$$\frac{1}{2}\frac{d}{dt} \| u_n \|^2_{L^2(\Omega)} + m \| u_n \|^p_{W_0^{1,p}(\Omega)} - \mu \| u_n \|^2_{L^2(\Omega)} \le c_1 |\Omega| + \int_\Omega g u_n dx.$$

Putting the above inequality together with (2.2), (2.3) and (2.4), there exist two positive constants C_1, C_2 such that

$$\frac{d}{dt}\| u_n \|^2_{L^2(\Omega)} + C_1 \| u_n \|^p_{W_0^{1,p}(\Omega)} \le C_2.$$

Integrating from 0 to t, $0 < t \le T_n$, we get

$$\| u_n(t) \|^2_{L^2(\Omega)} + C_1 \int_0^t \| u_n(s) \|^p_{W_0^{1,p}(\Omega)} ds \le \| u_0 \|^2_{L^2(\Omega)} + C_2 T.$$

This implies that $\{u_n\}$ is bounded in $L^\infty(0,T_n;L^2(\Omega))$ and in $L^p(0,T_n;W_0^{1,p}(\Omega))$. In particular, we see that $\| u_n(t) \|_{L^2(\Omega)}$ remains bounded in time. Therefore, we can extend the approximate solution to the whole interval $[0,T]$.

On the other hand, for a.e. $t \in (0,T)$,

$$Au_n(t) = -\mathrm{div}\left(a\left(\| \nabla u_n(t) \|^p_{L^p(\Omega)} \right) | \nabla u_n(t) |^{p-2} \nabla u_n(t) \right)$$

defines an element of $W^{-1,q}(\Omega)$ by

$$\langle Au_n(t), v \rangle = a\left(\| \nabla u_n(t) \|^p_{L^p(\Omega)} \right) \int_\Omega |\nabla u_n(t)|^{p-2} \nabla u_n(t) \cdot \nabla v dx,$$

for all $w \in W_0^{1,p}(\Omega)$. Using (1.2) and the boundedness of $\{u_n\}$ in $L^p(0,T;W_0^{1,p}(\Omega))$, we deduce that $\{Au_n\}$ is bounded in $L^q(0,T;W^{-1,q}(\Omega))$ since

$$\begin{aligned}
& \left| \int_0^T \langle -\mathrm{div}(a(\| \nabla u_n \|^p_{L^p(\Omega)}) | \nabla u_n |^{p-2} \nabla u_n), v \rangle dt \right| \\
= \; & \left| \int_{\Omega_T} a(\| \nabla u_n \|^p_{L^p(\Omega)}) | \nabla u_n |^{p-2} \nabla u_n \cdot \nabla v dxdt \right| \\
\le \; & M \int_{\Omega_T} |\nabla u_n|^{p-1} | \nabla v | dxdt \\
\le \; & M \| u_n \|^{p/q}_{L^p(0,T;W_0^{1,p}(\Omega))} \| v \|_{L^p(0,T;W_0^{1,p}(\Omega))}
\end{aligned}$$

for any $v \in L^p(0,T;W_0^{1,p}(\Omega))$. We now prove that $\{ f(u_n) \}$ is bounded in $L^1(\Omega_T)$. It follows from (1.2), (2.6) and (2.4) that

$$\frac{1}{2}\frac{d}{dt}\| u_n \|^2_{L^2(\Omega)} + \int_\Omega f(u_n) u_n dx \le C\left(m, p, q, \lambda_1, |\Omega| \right) \| g \|^q_{L^2(\Omega)}.$$

Integrating from 0 to T, we obtain

$$\frac{1}{2}\| u_n(T) \|^2_{L^2(\Omega)} + \int_{\Omega_T} f(u_n) u_n dxdt \le \frac{1}{2}\| u_n(0) \|^2_{L^2(\Omega)} + TC(m, p, q, \lambda_1, |\Omega|) \| g \|^q_{L^2(\Omega)}.$$

Hence

$$\int_{\Omega_T} f(u_n) u_n dxdt \le \frac{1}{2}\| u_0 \|^2_{L^2(\Omega)} + TC(m, p, q, \lambda_1, |\Omega|) \| g \|^q_{L^2(\Omega)}. \tag{2.7}$$

Setting $h(u_n) = f(u_n) - f(0) + \nu u_n$ with $\nu > \ell$. By (1.5), it implies that $h(s)s \ge 0$ for all $s \in \mathbb{R}$. Therefore, we deduce from (2.7) and the boundedness of $\{u_n\}$ in $L^\infty(0,T;L^2(\Omega))$ that

$$
\begin{aligned}
\int_{\Omega_T} |h(u_n)|\,dxdt &\le \int_{\Omega_T\cap\{|u_n|>1\}} |h(u_n)u_n|\,dxdt + \int_{\Omega_T\cap\{|u_n|\le 1\}} |h(u_n)|\,dxdt \\
&\le \int_{\Omega_T} h(u_n)u_n dxdt + \sup_{|s|\le 1}|h(s)||\Omega_T| \\
&= \int_{\Omega_T} f(u_n)u_n dxdt + \nu\int_{\Omega_T} |u_n|^2\,dxdt + |f(0)|\int_{\Omega_T} |u_n|\,dxdt \\
&\quad + \sup_{|s|\le 1}|h(s)||\Omega_T| \\
&\le C_3.
\end{aligned}
$$

This means that $\{h(u_n)\}$ is bounded in $L^1(\Omega_T)$, and so is $\{f(u_n)\}$. We rewrite the first equation of (1.1) in $L^q(0,T;W^{-1,q}(\Omega))+L^1(\Omega_T)$ as

$$
u_{nt} = g + \operatorname{div}(a(\|\nabla u_n\|^p_{L^p(\Omega)})|\nabla u_n|^{p-2}\nabla u_n) - f(u_n). \tag{2.8}
$$

Therefore, $\{u_{nt}\}$ is bounded in

$$
L^q(0,T;W^{-1,q}(\Omega))+L^1(\Omega_T) \subset L^1(0,T;W^{-1,q}(\Omega)+L^1(\Omega)).
$$

Since $W_0^{1,p}(\Omega)\cap L^\infty(\Omega) \subset\subset L^2(\Omega) \subset W^{-1,q}(\Omega)+L^1(\Omega),$ by the Aubin-Lions lemma, we see that $\{u_n\}$ is compact in $L^2(0,T;L^2(\Omega))$. Therefore, there is an a.e. convergent subsequence in Ω_T. Applying a diagonalization procedure and using Lemma 1.3 in [19, p. 12], we obtain (up to a subsequence) that

$$
\begin{aligned}
&u_n \rightharpoonup u \text{ in } L^p(0,T;W_0^{1,p}(\Omega)),\\
&u_n \to u \text{ in } L^2(0,T;L^2(\Omega)),\\
&u_{nt} \rightharpoonup u_t \text{ in } L^q(0,T;W^{-1,q}(\Omega))+L^1(\Omega_T),\\
&u_n(T) \to u(T) \text{ in } L^2(\Omega),
\end{aligned}
$$

and

$$
-\operatorname{div}(a\left(\|\nabla u_n\|^p_{L^p(\Omega)}\right)|\nabla u_n|^{p-2}\nabla u_n) \rightharpoonup -\chi \text{ in } L^q(0,T;W^{-1,q}(\Omega)). \tag{2.9}
$$

We now pass to the limit in the nonlinear term. From (1.5) we see that $h(\cdot)$ is a strictly increasing function. Moreover, using (2.7) we have

$$
\begin{aligned}
\int_{\Omega_T} h(u_n(t))u_n(t)dxdt &\le \frac{1}{2}\|u_n(0)\|^2_{L^2(\Omega)} + TC(m,p,q,\lambda_1,|\Omega|)\|g\|^q_{L^2(\Omega)} \\
&\quad + \frac{|f(0)|^2}{2}|\Omega|T + (\frac{1}{2}+\nu)\left(\|u_n(0)\|^2_{L^2(\Omega)} + C_2T\right).
\end{aligned}
$$

Since $u_n \to u$ strongly in $L^2(0,T;L^2(\Omega))$, then up to a subsequence, we have $u_n \to u$ a.e. in Ω_T. Applying Lemma 6.1 in [16], we obtain that $h(u)\in L^1(\Omega_T)$ and for all test function $\varphi \in C_0^\infty([0,T];W_0^{1,p}(\Omega)\cap L^\infty(\Omega))$,

$$\int_{\Omega_T} h(u_n)\varphi dxdt \to \int_{\Omega_T} h(u)\varphi dxdt \text{ as } n \to \infty.$$

Hence, $f(u) \in L^1(\Omega_T)$ and for all $\varphi \in C_0^\infty([0,T]; W_0^{1,p}(\Omega) \cap L^\infty(\Omega))$,

$$\int_{\Omega_T} f(u_n)\varphi dxdt \to \int_{\Omega_T} f(u)\varphi dxdt \text{ as } n \to \infty.$$

Now, passing to the limit in (2.8), one has in the distribution sense

$$u_t - \chi + f(u) = g. \tag{2.10}$$

It remains to prove that $\chi = Au$. To do this, integrating (2.6) from 0 to T we obtain

$$\int_0^T a\left(\| \nabla u_n \|^p_{L^p(\Omega)}\right)\int_\Omega |\nabla u_n|^p \, dxdt = \int_{\Omega_T} gu_n dxdt - \int_{\Omega_T} f(u_n)u_n dxdt + \frac{\| u_n(0) \|^2_{L^2(\Omega)}}{2} - \frac{\| u_n(T) \|^2_{L^2(\Omega)}}{2}.$$

Since $\lim\limits_{n\to\infty} \| u_n(T) \|^2_{L^2(\Omega)} = \| u(T) \|^2_{L^2(\Omega)}$ and $\lim\limits_{n\to\infty} \| u_n(0) \|^2_{L^2(\Omega)} = \| u_0 \|^2_{L^2(\Omega)}$, we deduce that

$$\lim_{n\to\infty}\int_0^T a\left(\| \nabla u_n \|^p_{L^p(\Omega)}\right)\int_\Omega |\nabla u_n|^p \, dxdt = \int_{\Omega_T} gudxdt - \int_{\Omega_T} f(u)udxdt + \frac{\| u_0 \|^2_{L^2(\Omega)}}{2} - \frac{\| u(T) \|^2_{L^2(\Omega)}}{2}. \tag{2.11}$$

Going back to (2.1), we have

$$\int_{\Omega_T} \left(a\left(\| \nabla u_n \|^p_{L^p(\Omega)}\right) |\nabla u_n|^{p-2} \nabla u_n - a\left(\| \nabla v \|^p_{L^p(\Omega)}\right) |\nabla v|^{p-2} \nabla v\right) \cdot \nabla(u_n - v) dxdt \ge 0$$

for all $v \in L^p(0,T; W_0^{1,p}(\Omega))$. Thus, taking limit leads to

$$\lim_{n\to\infty}\int_0^T a\left(\| \nabla u_n \|^p_{L^p(\Omega)}\right)\int_\Omega |\nabla u_n|^p \, dxdt + \int_0^T \langle \chi, v\rangle dt - \int_{\Omega_T} a\left(\| \nabla v \|^p_{L^p(\Omega)}\right) |\nabla v|^{p-2} \nabla v \cdot \nabla(u - v) dxdt \ge 0.$$

Putting this with (2.11), we have

$$\int_{\Omega_T} gudxdt - \int_{\Omega_T} f(u)udxdt + \frac{\| u_0 \|^2_{L^2(\Omega)}}{2} - \frac{\| u(T) \|^2_{L^2(\Omega)}}{2} + \int_0^T \langle \chi, v\rangle dt - \int_{\Omega_T} a\left(\| \nabla v \|^p_{L^p(\Omega)}\right) |\nabla v|^{p-2} \nabla v \cdot \nabla(u - v) dxdt \ge 0. \tag{2.12}$$

We see that $f(u) \in L^1(\Omega_T)$ and u does not belong to $W_0^{1,p}(\Omega) \cap L^\infty(\Omega)$. Therefore, u cannot be chosen as a test function in (2.10). We will use some ideas in [17]. Let $B_k : \mathbb{R} \to \mathbb{R}$ be the truncated function defined by

$$B_k(s) = \begin{cases} k & \text{if } \quad s > k, \\ s & \text{if } \quad |s| \le k, \\ -k & \text{if } \quad s < -k. \end{cases}$$

We construct the following Nemytskii mapping

$$\hat{B}_k : W_0^{1,p}(\Omega)\cap L^\infty(\Omega) \to W_0^{1,p}(\Omega)\cap L^\infty(\Omega)$$
$$v \ \mapsto \hat{B}_k(v)(x) = B_k(v(x)).$$

It follows from Lemma 2.3 in [17] that $\| \hat{B}_k(v)-v \|_{W_0^{1,p}(\Omega)\cap L^\infty(\Omega)} \to 0$ as $k\to\infty$. We now can test (2.10) by $\hat{B}_k(u)$. Multiplying (2.10) by $\hat{B}_k(u)$, then integrating from ε to T, we have

$$-\int_\varepsilon^T \langle \chi, \hat{B}_k(u)\rangle dt = \int_\varepsilon^T\int_\Omega g\hat{B}_k(u)dxdt - \int_\varepsilon^T\int_\Omega h(u)\hat{B}_k(u)dxdt$$
$$+\int_\varepsilon^T\int_\Omega (f(0)-\nu u)\hat{B}_k(u)dxdt + \int_\Omega u(\varepsilon)\hat{B}_k(u)(\varepsilon)dx$$
$$-\int_\Omega u(T)\hat{B}_k(u)(T)dx + \frac{1}{2}\| \hat{B}_k(u)(T)\|^2_{L^2(\Omega)} - \frac{1}{2}\| \hat{B}_k(u)(\varepsilon)\|^2_{L^2(\Omega)}.$$

Passing to the limit as $k\to\infty$ we have

$$-\int_\varepsilon^T \langle \chi, u\rangle dt = \int_\varepsilon^T\int_\Omega gudxdt - \lim_{k\to\infty}\int_\varepsilon^T\int_\Omega h(u)\hat{B}_k(u)dxdt$$

$$+\int_\varepsilon^T\int_\Omega (f(0)-\nu u)udxdt + \frac{1}{2}\| u(\varepsilon)\|^2_{L^2(\Omega)} - \frac{1}{2}\| u(T)\|^2_{L^2(\Omega)}, \tag{2.13}$$

where due to the nondecreasing of $\{h(u)\hat{B}_k(u)\}_{k=1}^\infty$ and $\hat{B}_k(u)\to u$ in $C([0,T];L^2(\Omega))$, it follows from the monotone convergence theorem that

$$\lim_{k\to\infty}\int_\varepsilon^T\int_\Omega h(u)\hat{B}_k(u)dxdt = \int_\varepsilon^T\int_\Omega h(u)udxdt.$$

We deduce from (2.13) by passing to the limit as $\varepsilon\to 0$ that

$$-\int_0^T \langle \chi, u\rangle dt = \int_{\Omega_T} gudxdt - \int_{\Omega_T} f(u)udxdt + \frac{\| u_0\|^2_{L^2(\Omega)}}{2} - \frac{\| u(T)\|^2_{L^2(\Omega)}}{2}. \tag{2.14}$$

In view of (2.12) and (2.14), we have

$$\int_0^T \langle \chi - \operatorname{div}(a\left(\| \nabla v\|^p_{L^p(\Omega)}\right)|\nabla v|^{p-2}\nabla v), u-v\rangle dt \le 0, \forall v\in L^p(0,T;W_0^{1,p}(\Omega)).$$

Choosing $v = u-\delta\varphi$, we deduce that

$$\int_0^T \langle \chi - \operatorname{div}(a(\|\nabla(u-\delta\varphi)\|^p_{L^p(\Omega)})|\nabla(u-\delta\varphi)|^{p-2}\nabla(u-\delta\varphi)), \varphi\rangle dt \le 0, \quad \text{if } \delta>0,$$
$$\int_0^T \langle \chi - \operatorname{div}(a(\|\nabla(u-\delta\varphi)\|^p_{L^p(\Omega)})|\nabla(u-\delta\varphi)|^{p-2}\nabla(u-\delta\varphi)), \varphi\rangle dt \ge 0, \quad \text{if } \delta<0,$$

for all $\varphi\in L^p(0,T;W_0^{1,p}(\Omega))$. Letting $\delta\to 0$, we get

$$\int_0^T \langle \chi - \operatorname{div}(a\left(\| \nabla u\|^p_{L^p(\Omega)}\right)|\nabla u|^{p-2}\nabla u), \varphi\rangle dt = 0, \forall \varphi\in L^p(0,T;W_0^{1,p}(\Omega)).$$

This implies that $\chi = \operatorname{div}\left(a\left(\| \nabla u \|_{L^p(\Omega)}^p\right) |\nabla u|^{p-2} \nabla u\right)$ in $L^q(0,T;W^{-1,q}(\Omega))$, which completes the proof of existence.

ii) Uniqueness and continuous dependence on the initial data

Let u, v be two weak solutions of (1.1) with initial data u_0, $v_0 \in L^2(\Omega)$, respectively. Then $w = u - v$ satisfies

$$\begin{cases} w_t - \operatorname{div}(a\left(\| \nabla u \|_{L^p(\Omega)}^p\right) |\nabla u|^{p-2} \nabla u) & + \operatorname{div}(a\left(\| \nabla v \|_{L^p(\Omega)}^p\right) |\nabla v|^{p-2} \nabla v) \\ & + f(u) - f(v) = 0, \\ w(0) = u_0 - v_0. \end{cases} \tag{2.15}$$

Multiplying the first equation in (2.15) by $\hat{B}_k(w)$, then integrating from ε to t, we obtain

$$\begin{aligned} &\int_\varepsilon^t \int_\Omega \frac{d}{ds}(w(s)\hat{B}_k(w)(s))dxds - \int_\varepsilon^t \int_\Omega w \frac{d}{ds}(\hat{B}_k(w)(s))dxds \\ &+ \int_\varepsilon^t \int_\Omega \left(a\left(\| \nabla u \|_{L^p(\Omega)}^p\right) |\nabla u|^{p-2} \nabla u - a\left(\| \nabla v \|_{L^p(\Omega)}^p\right) |\nabla v|^{p-2} \nabla v \right) \cdot \nabla(\hat{B}_k(w)(s))dxds \\ &+ \int_\varepsilon^t \int_\Omega (f(u) - f(v))\hat{B}_k(w)(s)dxds = 0. \end{aligned} \tag{2.16}$$

Since $w \frac{d}{dt}\hat{B}_k(w) = \frac{1}{2}\frac{d}{dt}(\hat{B}_k(w))^2$, we deduce from (2.1) and (1.5) by passing (2.16) to the limit as $k \to \infty$ and $\varepsilon \to 0$ that

$$\| w(t) \|_{L^2(\Omega)}^2 \le \| w(0) \|_{L^2(\Omega)}^2 + 2\ell \int_0^t \| w(s) \|_{L^2(\Omega)}^2 ds.$$

An application of the Gronwall inequality of integral form leads to

$$\| w(t) \|_{L^2(\Omega)}^2 \le \| w(0) \|_{L^2(\Omega)}^2 e^{2\ell t}, \text{ for all } t \in (0,T).$$

This implies the desired result.

3. Existence of global attractors

Theorem 2.1 allows us to construct a continuous (nonlinear) semigroup $S(t): L^2(\Omega) \to L^2(\Omega)$ associated to problem (1.1) as follows

$$S(t)u_0 := u(t),$$

where $u(t)$ is the unique global weak solution of (1.1) with the initial datum u_0.

3.1. Global attractor in $L^2(\Omega)$.

We first prove the following lemma.

Lemma 3.1. *The semigroup $\{S(t)\}_{t\ge 0}$ has a bounded absorbing set in $L^2(\Omega)$.*

Proof. From the first equation in (1.1), taking the inner product with u , we have

$$\frac{1}{2}\frac{d}{dt}\| u\|^2_{L^2(\Omega)} + a\left(\| \nabla u\|^p_{L^p(\Omega)}\right)\| u\|^p_{W_0^{1,p}(\Omega)} + \int_\Omega f(u)udx = \int_\Omega gudx. \tag{3.1}$$

In the case $p=2$, it follows from (1.2) and (1.4) that

$$\frac{1}{2}\frac{d}{dt}\| u\|^2_{L^2(\Omega)} + (m\lambda_1-\mu)\| u\|^2_{L^2(\Omega)} \le c_1|\Omega| + \int_\Omega gudx.$$

Since $m\lambda_1-\mu>0$, by the Young inequality, we obtain

$$\frac{d}{dt}\| u\|^2_{L^2(\Omega)} + (m\lambda_1-\mu)\| u\|^2_{L^2(\Omega)} \le 2c_1|\Omega| + \frac{1}{m\lambda_1-\mu}\| g\|^2_{L^2(\Omega)} .$$

In the case $p>2$, we deduce from (1.2) and (1.4) that

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\| u\|^2_{L^2(\Omega)} + \frac{1}{2}\| u\|^2_{L^2(\Omega)} + m\lambda_1\| u\|^p_{L^p(\Omega)} &\le \frac{1}{2}\| u\|^2_{L^2(\Omega)} - \int_\Omega f(u)udx + \int_\Omega gudx\\ &\le (\mu+1)\| u\|^2_{L^2(\Omega)} + c_1|\Omega| + \frac{1}{2}\| g\|^2_{L^2(\Omega)} .\end{aligned}$$

Moreover, there exists a positive constant C_5 such that

$$-m\lambda_1|s|^p + (\mu+1)s^2 \le C_5.$$

Thus, in both cases we have

$$\frac{d}{dt}\| u\|^2_{L^2(\Omega)} + \| u\|^2_{L^2(\Omega)} \le 2(C_5+c_1)|\Omega| + \| g\|^2_{L^2(\Omega)} .$$

Applying the Gronwall inequality, we get

$$\| u(t)\|^2_{L^2(\Omega)} \le \| u_0\|^2_{L^2(\Omega)} e^{-R_1 t} + R_2(1-e^{-R_1 t}), \tag{3.2}$$

where $R_1 = m\lambda_1-\mu, R_2 = 2c_1|\Omega| + \dfrac{1}{m\lambda_1-\mu}\| g\|^2_{L^2(\Omega)}$ if $p=2$ and $R_1=1,$ $R_2 = 2(C_5+c_1)|\Omega| + \| g\|^2_{L^2(\Omega)}$ if $p>2$. Therefore,

$$\| u(t)\|^2_{L^2(\Omega)} \le \rho_0 \tag{3.3}$$

for all $t\ge T_0 = T_0(\| u_0\|^2_{L^2(\Omega)})$, where $\rho_0 = 2R_2$ is independent of u_0 .

Lemma 3.2. *The semigroup* $\{S(t)\}_{t\ge 0}$ *has a bounded absorbing set in* $W_0^{1,p}(\Omega)$.

Proof. Multiplying the first equation in (1.1) by $-\Delta_p u$ and integrating by parts, we obtain

$$\frac{1}{p}\frac{d}{dt}\| u\|^p_{W_0^{1,p}(\Omega)} + a\left(\| \nabla u\|^p_{L^p(\Omega)}\right)\| \Delta_p u\|^2_{L^2(\Omega)} = -\int_\Omega f'(u)|\nabla u|^p dx - \int_\Omega g\Delta_p udx.$$

Using (1.2), (1.4) and the Cauchy inequality, we deduce that

$$\frac{d}{dt}\| u\|^p_{W_0^{1,p}(\Omega)} \le \ell p\| u\|^p_{W_0^{1,p}(\Omega)} + \frac{p}{4m}\| g\|^2_{L^2(\Omega)} . \tag{3.4}$$

On the other hand, integrating (3.1) from t to $t+1$ and using (1.3) together with the Cauchy inequality, we have

$$\int_t^{t+1} a\left(\| \nabla u(s) \|_{L^p(\Omega)}^p\right) \| u(s) \|_{W_0^{1,p}(\Omega)}^p \, ds + \frac{1}{2} \| u(t+1) \|_{L^2(\Omega)}^2$$
$$\leq \left(\mu + \frac{1}{2} \right) \int_t^{t+1} \| u(s) \|_{L^2(\Omega)}^2 \, ds + \frac{1}{2} \| u(t) \|_{L^2(\Omega)}^2 + c_1 |\Omega| + \frac{1}{2} \| g \|_{L^2(\Omega)}^2 .$$

In view of (3.2) and (1.2), we get the following estimate

$$\int_t^{t+1} \| u(s) \|_{W_0^{1,p}(\Omega)}^p \, ds \leq \frac{1}{m} \left((\mu + \frac{1}{2}) \rho_0 + c_1 |\Omega| + \frac{1}{2} \| g \|_{L^2(\Omega)}^2 \right), \tag{3.5}$$

for all $t \geq T_0$. As an application of the uniform Gronwall inequality, we deduce from (3.4) and (3.5) that

$$\| u(t) \|_{W_0^{1,p}(\Omega)}^p \leq \rho_1, \tag{3.6}$$

for all $t \geq T_1 = T_0 + 1$, where $\rho_1 = \left[\frac{2\mu + 1}{2m} \rho_0 + c_1 |\Omega| + \frac{p+2}{4m} \| g \|_{L^2(\Omega)}^2 \right] e^{\ell p}$.

The following theorem is a direct consequence of Lemma 3.2 and the compactness of the embedding $W_0^{1,p}(\Omega) \subset\subset L^2(\Omega)$.

Theorem 3.1. *Under the assumptions* **(H1) - (H3)**, *the semigroup* $\{S(t)\}_{t \geq 0}$ *generated by problem (1.1) has a compact global attractor* $\mathcal{A}_2$ *in* $L^2(\Omega)$.

3.2. Global attractor in $W_0^{1,p}(\Omega)$

In this subsection we will prove the existence of a global attractor in $W_0^{1,p}(\Omega)$ under the following additional assumption

(H1bis) a is a continuously differentiable and nondecreasing function satisfying **(H1)**.

We first define the following subset

$$\mathbb{B}_R = \left\{ u \in L^2(\Omega) : \ \| u \|_{W_0^{1,p}(\Omega)} + \| \Delta_p u \|_{L^2(\Omega)} \leq R \right\}.$$

We see that $\mathbb{B}_R$ is the subset of the domain of Δ_p acting on $L^2(\Omega)$. Moreover, it is is precompact in $W_0^{1,p}(\Omega)$ (see [16, Remark 4.3]). We have the following important lemma.

Lemma 3.3. *Under the assumptions* **(H1bis), (H2)** *and* **(H3).** *For* $R > 0$ *sufficiently large,* $\mathbb{B}_R$ *is an absorbing set for the semigroup* $S(t)$ *acting on* $L^2(\Omega)$ *(hence absorbing on* $W_0^{1,p}(\Omega)$*).*

Proof. It is enough to prove that the bounded absorbing set in Lemma 3.2 is absorbed into $\mathbb{B}_R$ for some $R > 0$. Indeed, we denote $v = u_t$. By differentiating the first equation in (1.1) in time, we obtain

$$
\begin{aligned}
& v_t - \operatorname{div}\Big(a\big(\| \nabla u \|^p_{L^p(\Omega)} \big) | \nabla u |^{p-2} \nabla v \Big) \\
& \quad -(p-2)\operatorname{div}\Big(a\big(\| \nabla u \|^p_{L^p(\Omega)} \big) | \nabla u |^{p-4} (\nabla u \cdot \nabla v) \nabla u \Big) \\
& \quad - p \operatorname{div}\Big(a'\big(\| \nabla u \|^p_{L^p(\Omega)} \big) \int_\Omega |\nabla u|^{p-2} (\nabla u \cdot \nabla v) dx \, | \nabla u |^{p-2} \nabla u \Big) + f'(u) v = 0.
\end{aligned}
$$

Taking the inner product of the above equality with v and using (1.4), one gets

$$
\begin{aligned}
& \frac{1}{2} \| v \|^2_{L^2(\Omega)} + a\big(\| \nabla u \|^p_{L^p(\Omega)} \big) \int_\Omega |\nabla u|^{p-2} |\nabla v|^2 \, dx \\
& \qquad + (p-2) a\big(\| \nabla u \|^p_{L^p(\Omega)} \big) \int_\Omega |\nabla u|^{p-4} (\nabla u \cdot \nabla v)^2 dx \\
& \qquad + p \, a'\big(\| \nabla u \|^p_{L^p(\Omega)} \big) \Big(\int_\Omega |\nabla u|^{p-2} (\nabla u \cdot \nabla v) dx \Big)^2 \le \ell \| v \|^2_{L^2(\Omega)} .
\end{aligned}
$$

By assumption **(H1bis)**, it follows from the last inequality that

$$
\frac{d}{dt} \| v \|^2_{L^2(\Omega)} \le 2\ell \| v \|^2_{L^2(\Omega)} . \tag{3.7}
$$

On the other hand, multiplying the first equation in \eqref{eq:1.1} by u_t, we get

$$
\| u_t \|^2_{L^2(\Omega)} + \frac{1}{p} a\big(\| \nabla u \|^p_{L^p(\Omega)} \big) \frac{d}{dt} \| u \|^p_{W_0^{1,p}(\Omega)} + \int_\Omega f(u) u_t dx - \int_\Omega g u_t dx = 0.
$$

We can rewrite this equality as follows

$$
\begin{aligned}
& \| u_t \|^2_{L^2(\Omega)} + \frac{d}{dt} \Big[\frac{1}{p} a\big(\| \nabla u \|^p_{L^p(\Omega)} \big) \int_\Omega |\nabla u|^p \, dx + \int_\Omega F(u) dx - \int_\Omega g u dx \Big] \\
& \qquad = \frac{1}{p} a'\big(\| \nabla u \|^p_{L^p(\Omega)} \big) \| \nabla u \|^p_{L^p(\Omega)} \frac{d}{dt} \| \nabla u \|^p_{L^p(\Omega)} .
\end{aligned}
$$

Setting $L = \sup\limits_{0 \le s \le \rho_1} | a'(s) |$.

In view of **(H1bis)**, (3.4) and (3.6), we deduce that

$$
\begin{aligned}
& \| u_t \|^2_{L^2(\Omega)} + \frac{d}{dt} \Big[\frac{1}{p} a\big(\| \nabla u \|^p_{L^p(\Omega)} \big) \| u \|^p_{W_0^{1,p}(\Omega)} + \int_\Omega F(u) dx - \int_\Omega g u dx \Big] \\
& \le L \rho_1 \Big(\ell \rho_1 + \frac{1}{4m} \| g \|^2_{L^2(\Omega)} \Big).
\end{aligned} \tag{3.8}
$$

On the other hand, integrating (3.1) from t to $t+1$ and using (3.3) leads to

$$
\int_t^{t+1} \Big[a\big(\| \nabla u \|^p_{L^p(\Omega)} \big) \| u \|^p_{W_0^{1,p}(\Omega)} + \int_\Omega f(u) u dx - \int_\Omega g u dx \Big] ds \le \frac{\rho_0}{2},
$$

for all $t \ge T_0$. It follows from (1.5) that

$$
-\frac{\ell + 1}{2} u^2 - \frac{| f(0) |^2}{2} \le F(u) \le f(u) u + \frac{\ell}{2} u^2, \text{ for all } u \in \mathbb{R}.
$$

Hence, we have

$$\int_t^{t+1}\Big[\frac{1}{p}a\left(\|\nabla u\|^p_{L^p(\Omega)}\right)\|u\|^p_{W_0^{1,p}(\Omega)}+\int_\Omega F(u)dx-\int_\Omega gudx\Big]ds\le\frac{\ell+1}{2}\rho_0, \tag{3.9}$$

for all $t\ge T_0$. Using the uniform Gronwall inequality, it follows from (3.8) and (3.9) that

$$\frac{1}{p}a\left(\|\nabla u\|^p_{L^p(\Omega)}\right)\|u\|^p_{W_0^{1,p}(\Omega)}+\int_\Omega F(u)dx-\int_\Omega gudx\le\rho_2, \tag{3.10}$$

for all $t\ge T_2=T_1+1$, and $\rho_2=\frac{\ell+1}{2}\rho_0+L\rho_1\left(\ell\rho_1+\frac{1}{4m}\|g\|^2_{L^2(\Omega)}\right)$. Integrating (3.8) from t to $t+1$ and using (3.10), we infer that

$$\int_t^{t+1}\|u_t\|^2_{L^2(\Omega)}ds\le(\ell+1)\rho_0+3L\rho_1\left(\ell\rho_1+\frac{1}{4m}\|g\|^2_{L^2(\Omega)}\right),\text{ for all } t\ge T_2. \tag{3.11}$$

Using the uniform Gronwall inequality again, it follows from (3.7) and (3.11) that

$$\|u_t(t)\|^2_{L^2(\Omega)}\le\left[(\ell+1)\rho_0+3L\rho_1\left(\ell\rho_1+\frac{1}{4m}\|g\|^2_{L^2(\Omega)}\right)\right]e^{2\ell}, \tag{3.12}$$

for all $t\ge T_3=T_2+1$. On the other hand, multiplying the first equation in (1.1) by $-\Delta_p u$, using (1.4) and the Cauchy inequality, we obtain

$$\begin{aligned} a\left(\|\nabla u\|^p_{L^p(\Omega)}\right)\|\Delta_p u\|^2_{L^2(\Omega)} &= \int_\Omega u_t\Delta_p udx-\int_\Omega f'(u)|\nabla u|^p\,dx-\int_\Omega g\Delta_p udx \\ &\le \ell\|u\|^p_{W_0^{1,p}(\Omega)}+\frac{m}{2}\|\Delta_p u\|^2_{L^2(\Omega)}+\frac{1}{m}\|u_t\|^2_{L^2(\Omega)}+\frac{1}{m}\|g\|^2_{L^2(\Omega)}. \end{aligned}$$

The following estimate is obtained from (1.2), (3.3), (3.6) and (3.12),

$$\|\Delta_p u(t)\|^2_{L^2(\Omega)}\le\frac{2\ell}{m}\rho_1+\frac{2}{m^2}\rho_0+\frac{2}{m^2}\|g\|^2_{L^2(\Omega)},\text{ for all } t\ge T_3.$$

This combining with (3.6) implies the desired result.

By the similar arguments of Corollary 4.5 in [16], we get the following result.

Theorem 3.4. *Under the assumptions* **(H1bis), (H2)** *and* **(H3)**, *the semigroup* $\{S(t)\}_{t\ge0}$ *associated to problem (1.1) has a compact global attractor* $\mathcal{A}$ *in* $W_0^{1,p}(\Omega)$.

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